CS61B Lecture #20: Trees

A Recursive Structure

lly represent recursively defined, hierarchical objects
an one recursive subpart for each instance.

mples: expressions, sentences.

ns have definitions such as “an expression consists of a
two expressions separated by an operator.”

e search structures in which we recursively divide a set
disjoint subsets.

Formal Definitions

in a variety of flavors, all defined recursively:

e: A tree consists of a *label* value and zero or more
(or *children*), each of them a tree.

, alternative definition: A tree is a set of *nodes* (or
each of which has a label value and one or more *child*
ch that no node descends (directly or indirectly) from
ode is the *parent* of its children.

trees: A tree is either *empty* or consists of a node
a label value and an indexed sequence of zero or more
each a positional tree. If every node has two positions,
binary tree and the children are its *left and right sub-*
in, nodes are the parents of their non-empty children.

other varieties when considering graphs.

Tree Characteristics (I)

a tree is a non-empty node with no parent in that tree
ight be in some larger tree that contains that tree as
hus, every node is the root of a (sub)tree.

rity, or *degree* of a node (tree) is its number (maximum
hildren.

a *k-ary tree* each have at most k children.

has no children (no non-empty children in the case of
ees).

Tree Characteristics (II)

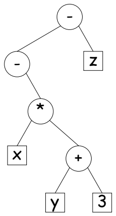
f a node in a tree is the largest distance to a leaf. That
s height 0 and a non-empty tree's height is one more
imum height of its children. The height of a tree is the
root.

f a node in a tree is the distance to the root of that
, in a tree whose root is R, R itself has depth 0 in R,
$\neq R$ is in the tree with root R, then its depth is one
its parent's.

A Tree Type, 61A Style

```
le Tree whose labels are of type LABEL. */
ree<Label> {

arnings("unchecked")  // Explained in a later lecture
e(Label label, Tree<Label>... children) {
 = label;
= new ArrayList<>(Arrays.asList(children));


 arity() { return _kids.size(); }

el label() { return _label; }

e<Label> child(int k) { return _kids.get(k); }

bel _label;
rayList<Tree<Label>> _kids;
```

der Traversal and Prefix Expressions



onvert **into** (- (- (* x (+ y 3))) z)

```
 toLisp(Tree<String> T) {
() == 0) return T.label();

  R = "(" + T.label();
. = 0; i < T.arity(); i += 1)
' + toLisp(T.child(i));
- ")";
```

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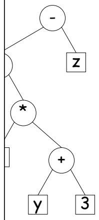
der Traversal and Postfix Expressions

ert



into x y 3 +:2 *:2 -:1 z -:2

```
 toPolish(Tree<String>  T)  {
: = "";
= 0; i < T.arity(); i += 1)
olish(T.child(i)) + " ";
String.format("%s:%d", T.label(), T.arity());
```

---

terative Depth-First Traversals

on conceals data: a *stack* of nodes (all the `T` arguments) xtra information. Can make the data explicit:

```
Traverse2(Tree<Label> T, Consumer<Tree<Label>> visit) {
abel>> work = new Stack<>();
;
.isEmpty()) {
> node = work.pop();
pt(node);
 = node.arity()-1; i >= 0; i -= 1)
ush(node.child(i));  // Why backward?
```

al takes the same $\Theta(\cdot)$ time as doing it recursively, and e $\Theta(\cdot)$ space.

ave substituted an explicit stack data structure (`work`) ilt-in execution stack (which handles function calls).

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undamental Operation: Traversal

tree means enumerating (some subset of) its nodes.

e recursively, because that is natural description.

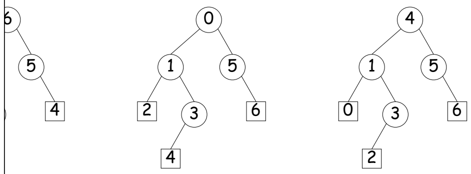
enumerated, we say they are *visited*.

orders for enumeration (+ variations):

visit node, traverse its children.

: traverse children, visit node.

traverse first child, visit node, traverse second child ees only).



rder Preorder inorder

---

der Traversal and Infix Expressions

ert

into ((-(x*(y+3)))-z)

To think about: how to get rid of all those parentheses.

```
 toInfix(Tree<String> T) {
() == 0) {
abel();
.arity() == 1) {
' T.label() + toInfix(T.child(0)) + ")";

' toInfix(T.child(0)) + T.label() + toInfix(T.child(1)) + ")";
```

---

neral Traversal: The Visitor Pattern

```
erTraverse(Tree<Label> T, Consumer<Tree<Label>> visit)

null) {
ccept(T);
 i = 0; i < T.arity(); i += 1)
derTraverse(T.child(i), visit);
```

unction.Consumer<AType> is a library interface that unction-like type with one void method, `accept`, which ument of type `AType`.

ava 8 lambda syntax, I can print all labels in the tree in h:

```
averse(myTree, T -> System.out.print(T.label() + " "));
```

adth-First Traversal Implemented

cation to iterative depth-first traversal gives breadth-
Just change the (LIFO) stack to a (FIFO) queue:

```
rstTraverse(Tree<Label> T, Consumer<Tree<Label>> visit) {
ree<Label>> work = new ArrayDeque<>();  // (Changed)
;
.isEmpty()) {
> node = work.remove();    // (Changed)
= null) {
accept(node);
nt i = 0; i < node.arity(); i += 1) // (Changed)
k.push(node.child(i));
```

eadth-First Traversal: Iterative Deepening

adth-first traversal used space proportional to the *width* which is $\Theta(N)$ for bushy trees, whereas depth-first kes $\lg N$ space on bushy trees.

preadth-first traversal in $\lg N$ space and $\Theta(N)$ time on

el, k, of the tree from 0 to lev, call `doLevel(T,k)`:

```
el(Tree T, int lev) {
= 0)


h non-null child, C, of T {
el(C, lev-1);
```

eadth-first traversal by repeated (truncated) depth-
als: *iterative deepening*.

`T, k)`, we skip (i.e., traverse but don't visit) the nodes k, and then visit at level k, but not their children.

Iterators for Trees

ators are not terribly convenient on trees.

deas from iterative methods.

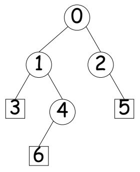
```
rderTreeIterator<Label> implements Iterator<Label> {
Stack<Tree<Label>> s = new Stack<Tree<Label>>();

reorderTreeIterator(Tree<Label> T) { s.push(T);  }

oolean hasNext() { return !s.isEmpty(); }
 next() {
abel> result = s.pop();
nt i = result.arity()-1; i >= 0; i -= 1)
sh(result.child(i));
 result.label();
```

t do I have to add to class `Tree` first?)

```
ring label : aTree) System.out.print(label + " ");
```

l-Order (Breadth-First) Traversal

erse all nodes at depth 0, then depth 1, etc:



Times

l algorithms have roughly the form of the `boom` example *ata Structures*—an exponential algorithm.

e role of M in that algorithm is played by the *height* of the number of nodes.

y to see that tree traversal is *linear:* $\Theta(N)$, where N nodes: Form of the algorithm implies that there is one root, and then one visit for every *edge* in the tree. node but the root has exactly one parent, and the root st be $N-1$ edges in any non-empty tree.

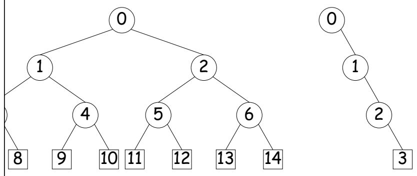
tree, is also one recursive call for each empty tree, but trees can be no greater than kN, where k is arity.

e (max # children is k), $h+1 \leq N \leq \frac{k^{h+1}-1}{k-1}$, where h is

$_k N) = \Omega(\lg N)$ and $h \in O(N)$.

lgorithms look at one child only. For them, worst-case ortional to the *height* of the tree—$\Theta(\lg N)$—assuming *bushy*—each level has about as many nodes as possible.

Iterative Deepening Time?



ght, N be # of nodes.

es traversed (i.e, # of calls, not counting null nodes).

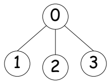
ee: 1 for level 0, 3 for level 1, 7 for level 2, 15 for level

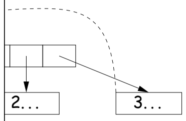
$(2^1-1)+(2^2-1)+\ldots+(2^{h+1}-1) = 2^{h+2}-h \in \Theta(N)$, $^{+1}-1$ for this tree.

t leaning) tree: 1 for level 0, 2 for level 2, 3 for level 3.

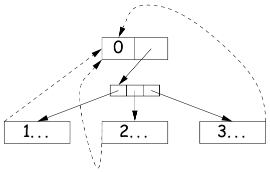
$(h+1)(h+2)/2 = N(N+1)/2 \in \Theta(N^2)$, since $N = h+1$ of tree.

Tree Representation

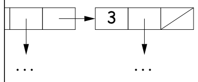




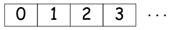
ed child pointers
parent pointers)



(b) Array of child pointers
(+ optional parent pointers)



sibling pointers



(d) breadth-first array
(complete trees)