

## Searching by "Generate and Test"

Considering the problem of searching a set of data stored in a data structure: "Is  $x \in S$ ?"

We *don't* have a set  $S$ , but know how to recognize what we find it: "Is there an  $x$  such that  $P(x)$ ?"

How to enumerate all possible candidates, can use approach *Generate and Test*: test all possibilities in turn.

Can be more clever: avoid trying things that won't work, e.g. pruning.

What if the set of possible candidates is infinite?

## General Recursive Algorithm

**isKnightMove** a sequence of knight moves starting at ROW, COL is all squares that have been hit already and up one square away from ENDROW, ENDCOL. B[i][j] is true if row i and column j have been hit on PATH so far. Return true if it succeeds, else false (with no change to PATH). Update B locally with PATH containing the starting square, and ending square (only) marked in B. \*/

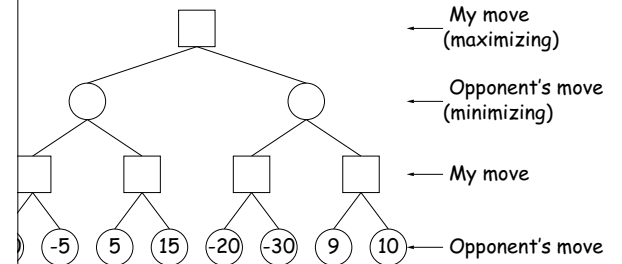
```

boolean isKnightMove(int row, int col, int endRow, int endCol, List path) {
    if (row == endRow && col == endCol) return true;
    for (int r = row - 2; r <= row + 2; r++) {
        for (int c = col - 1; c <= col + 1; c++) {
            if (r == row && c == col) continue;
            if (!B[r][c]) {
                B[r][c] = true; // Mark the square
                if (isKnightMove(r, c, endRow, endCol, path)) return true;
                B[r][c] = false; // Backtrack out of the move.
            }
        }
    }
    return false;
}

```

## Game Trees

View the space of possible continuations of the game as a tree. At each position, each edge a move.



The numbers at the bottom are the values of those final positions. Higher numbers are of more value to *my opponent*.

What is my move? What value can I get if my opponent plays as optimally?

## CS61B Lecture #23

Backtracking searches, game trees (DSIJ, Section 6.5)

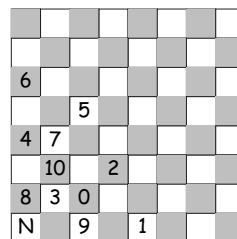
## Backtracking Search

Backtracking search is one way to enumerate all possibilities.

**Knight's Tour**. Find all paths a knight can travel on a chessboard such that it touches every square exactly once and ends at the starting square.

Below, the numbers indicate position numbers (knight starts at 0).

What if the knight is stuck; how to handle this?



## Another Kind of Search: Best Move

The problem of finding the *best* move in a two-person game. Assign a *heuristic value* to each possible move and pick the move with the highest *static valuation*.

We can use a variety of heuristics. Some examples of heuristics:

1. Maximal or minimal value to a won position (depending on the game).

2. Difference of black pieces – number of white pieces in checkers. 3. Sum of white piece values) – (weighted sum of black piece values), such as queen=9, rook=5, knight=bishop=3, pawn=1. 4. Number of pieces in strategic areas (center of board).

5. Avoid misleading. A move might give us more pieces, but set up a trap for a response from the opponent.

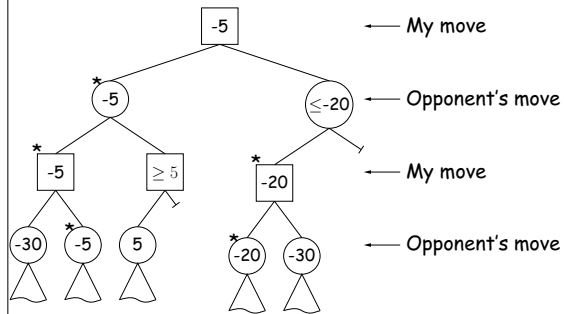
6. In a minimax search, look at *opponent's* possible moves, use the best result *for the opponent* as the value.

7. How do you have a great response to opponent's response?

8. How do you organize this sensibly?

## Alpha-Beta Pruning

See this tree as we search it.



At this position, I know that the opponent will not choose to already has a  $-5$  move).

At this position, my opponent knows that I will never choose (since I already have a  $-5$  move).

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## Overall Search Algorithm

whose move it is (maximizing player or minimizing player), for a move estimated to be optimal in one direction or

be exhaustive down to a particular depth in the game that, we guess values.

and  $\beta$  limits:

er does not care about exploring a position further after at its value will be larger than a position the minimizing already found, because the minimizing player will simply e a position with that larger value.

minimizing player won't explore a positions whose value in what the maximizing player can get ( $\alpha$ ).

maximizing player will find a move with the call

`Value(current position, search depth,  $-\infty$ ,  $+\infty$ )`

ayer:

`Value(current position, search depth,  $-\infty$ ,  $+\infty$ )`

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## Code for Searching (Maximizing Player)

```

return the minimax value of position POSN, searching up to
ahead, assuming it is the maximizing player's move.
is determined to be  $\leq$ ALPHA, then the function
by value  $\leq$ ALPHA, even if inaccurate. Likewise if the
ETA, it may return any value  $\geq$ BETA. Assumes ALPHA<BETA. */
return Value(Position posn, int depth, int alpha, int beta)
    
```

```

final position of the game || depth == 0)
return staticGuess(posn);
float f =  $-\infty$ ;
for all move, M, in position posn) {
    next = makeMove(posn, M);
    float b = minPlayerValue(next, depth-1, alpha, beta);
    if (b > bestSoFar) {
        bestSoFar = b;
        response = M;
    }
    alpha = max(alpha, bestSoFar);
    if (alpha >= beta)
        return bestSoFar;
    }
    }
    
```

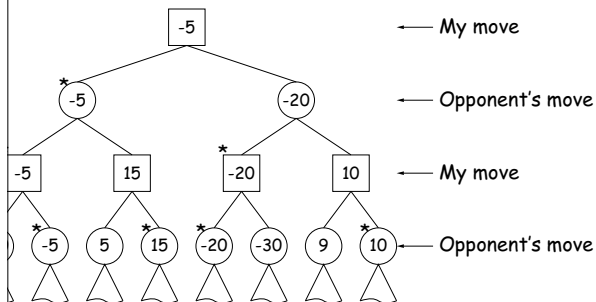
bestSoFar;

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## Game Trees, Minimax

space of possible continuations of the game as a tree. at a position, each edge a move.



See the values we guess for the positions (larger means better). Starred nodes would be chosen.

Choose child (next position) with maximum value; opponent chooses minimum value—the *minimax algorithm*.

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## Cutting off the Search

When you traverse game tree to the bottom, you'd be able to stop if it's possible.

Search is possible near the end of a game.

Typically, game trees tend to be either infinite or impossibly

large. We set a maximum *depth*, and use a heuristic static valuation at that depth.

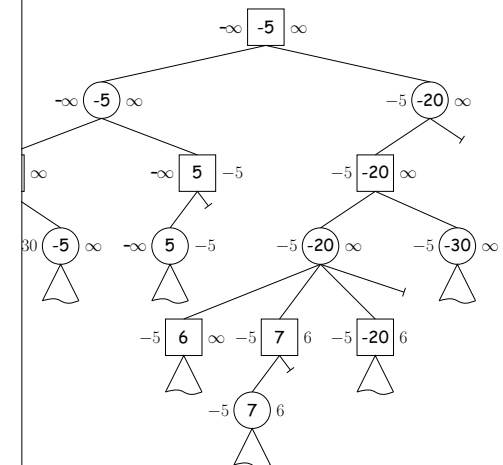
Instead, we use *iterative deepening*, repeating the search at increasing time is up.

More sophisticated searches are possible, however (take CS188).

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## Example Tree with Alpha and Beta Values



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## Code for Searching (Minimizing Player)

```
int minimax value of position POSN, searching up to
depth ahead, assuming it is the minimizing player's move. */
int minimax(Position posn, int depth, int alpha, int beta)
```

```
    // final position of the game || depth == 0
    int bestSoFar = staticGuess(posn);
    int r = +∞;
    for (all move, M, in position posn) {
        Position next = makeMove(posn, M);
        int response = maxPlayerValue(next, depth-1, alpha, beta);
        if (response < bestSoFar) {
            bestSoFar = response;
        }
        alpha = min(beta, bestSoFar);
        if (alpha >= beta)
            return bestSoFar;
    }
    return bestSoFar;
}
```

bestSoFar;