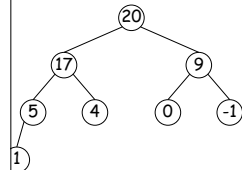


Priority Queues, Heaps

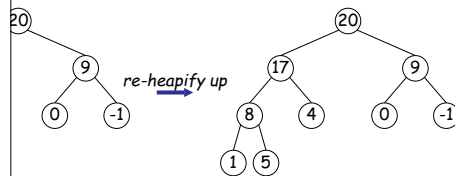
are defined by operations "add," "find largest," "remove largest," and "insert." They are used for scheduling long streams of actions to occur at various times and for sorting (keep removing largest). Implementation is the *heap*, a kind of tree. (The same term is used to describe the pool of storage an operator uses. Sorry about that.)

Example: Inserting into a simple heap

1 20

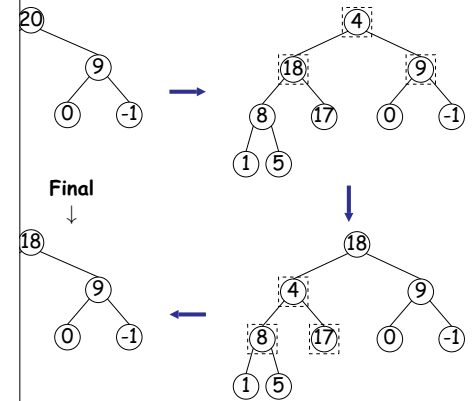


Dashed boxes show where heap property violated



Removing Largest from Heap

Algorithm: Move bottommost, rightmost node to top, then re-heapify as needed (swap offending node with larger child) to restore heap property.



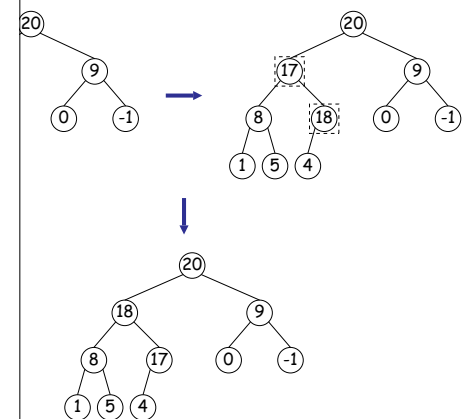
CS61B Lecture #24

Priority Queues (Data Structures §6.4, §6.5)
 Heaps (§6.2)
 Applications: SortedSet, Map, etc.

Heaps

A *heap* is a binary tree that enforces the *heap property*: Labels of *both* children of each node are less than or equal to the node's label. The root node has the largest label. This property enforces the binary search property, which allows us to keep tree operations always valid to put the smallest nodes anywhere at the leaf level of the tree. A heap can be made *nearly complete*: all but possibly the last level are full, and the last level has as many keys as possible. The time complexity of insertion of new value and deletion of largest value are both $O(\lg N)$ in worst case. A *min-heap* is basically the same, but with the minimum value at the root and children having larger values than their parents.

Heap insertion continued



Ranges

looked for specific items

sets, need an ordering anyway, and can also support looking for values.

perform some action on all values in a BST that are within in natural order):

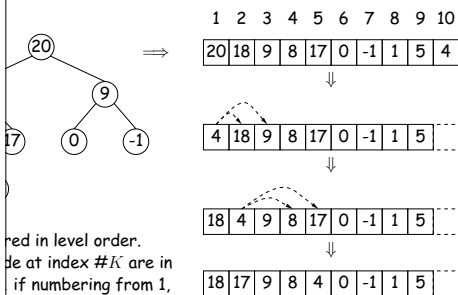
```

WHATTODO to all labels in T that are >= L and < U,
ending natural order. */
void visitRange(BST<String> T, String L, String U,
               Consumer<BST<String>> whatToDo) {
    if (whatToDo == null) {
        return;
    }
    int left = L.compareTo(T.label ()),
        right = U.compareTo(T.label ());
    if (left < 0) /* L < label */
        visitRange (T.left(), L, U, whatToDo);
    if (left <= 0 && right > 0) /* L <= label < U */
        whatToDo.accept(T);
    if (right > 0) /* label < U */
        visitRange (T.right (), L, U, whatToDo);
}
    
```

Heaps in Arrays

are nearly complete (missing items only at bottom level),
 arrays for compact representation.

removal from last slide (dashed arrows show children):



ordered in level order.
 elements at index #K are in
 range [K, 2K) if numbering from 1,
 range [K/2, K) if from 0.

Tree Sets and Range Queries in Java

TreeSet supports range queries with views of set:

headSet(U): subset of S that is < U.

tailSet(L): subset that is ≥ L.

subSet(L,U): subset that is ≥ L, < U.

Views modify S.

Operations, e.g., add to a headSet beyond U are disallowed.

Use a view to process a range:

```

String> fauna = new TreeSet<String>();
fauna.addAll ("axolotl", "elk", "dog", "hartebeest", "duck");
Set<String> item = fauna.subSet ("bison", "gnu");
item.forEach (item -> out.printf ("%s, ", item));
    
```

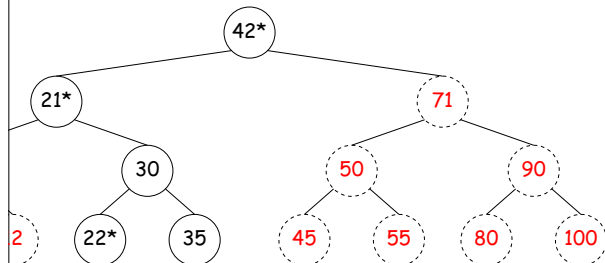
Output: dog, duck, elk, "

Time for Range Queries

A range query is $O(h + M)$, where h is height of tree, and M
 is number of data items that turn out to be in the range.

Traversing the tree below for all values $25 \leq x < 40$.

Nodes are never looked at. Starred nodes are looked at but
 their children are not. The h comes from the starred nodes; the M comes from
 the number of nodes in the range.



Example of Representation: BSTSet

Representation for

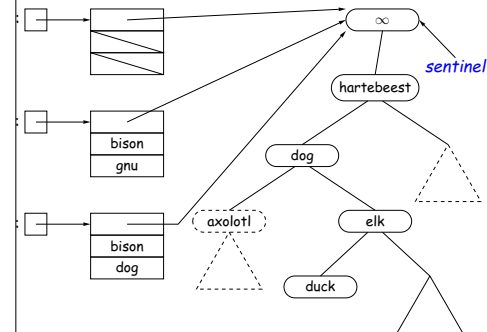
```

SortedSet<String> fauna = new BSTSet<String>(stuff);
SortedSet<String> subset1 = fauna.subSet("bison", "gnu");
SortedSet<String> subset2 = subset1.subSet("axolotl", "dog");
    
```

BST, plus

view.

Expensive!



TreeSet

TreeSet<T> requires either that T be Comparable,
 or provide a Comparator, as in:

```

String> rev_fauna = new TreeSet<String>(Collections.reverseOrder());
    
```

Comparator is a type of function object:

```

Comparator<T> {
    int compare(T left, T right);
}
    
```

Comparator<T> extends Comparable<T> is all

the reverseOrder comparator is defined like this:

```

Comparator<T> reverseOrder() {
    return (x, y) -> y.compareTo(x);
}
    
```

Comparing Search Structures

ms, k is #answers to query.

	Unordered List	Sorted Array	Bushy Search Tree	"Good" Hash Table	Heap
nd)	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(N)$
	$\Theta(1)$	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(\lg N)$
	$\Theta(N)$	$\Theta(k + \lg N)$	$\Theta(k + \lg N)$	$\Theta(N)$	$\Theta(N)$
	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(1)$
st	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(\lg N)$