CS61B Lecture #24

Today:

- Priority queues (*Data Structures* §6.4, §6.5)
- Range queries (§6.2)
- Java utilities: `SortedSet`, `Map`, etc.

Priority Queues, Heaps

- Priority queue: defined by operations "add," "find largest," "remove largest."
- Examples: scheduling long streams of actions to occur at various future times.
- Also useful for sorting (keep removing largest).
- Common implementation is the *heap*, a kind of tree.
- (Confusingly, this same term is used to described the pool of storage that the **new** operator uses. Sorry about that.)

Heaps

- A *max-heap* is a binary tree that enforces the

  Heap Property: Labels of *both* children of each node are less than node's label.

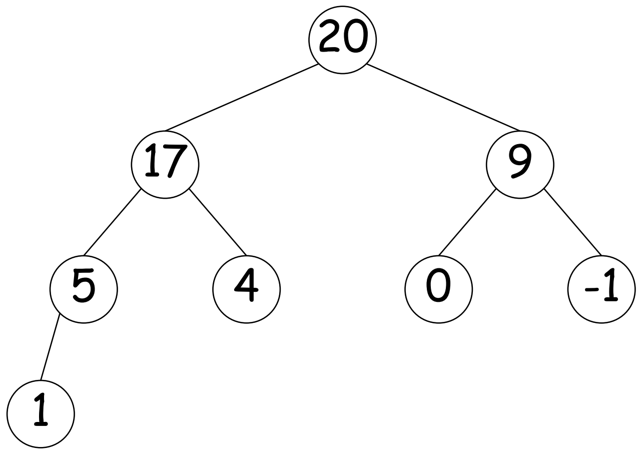
- So node at top has largest label.
- Looser than binary search property, which allows us to keep tree "bushy".
- That is, it's always valid to put the smallest nodes anywhere at the bottom of the tree.
- Thus, heaps can be made *nearly complete:* all but possibly the last row have as many keys as possible.
- As a result, insertion of new value and deletion of largest value always take time proportional to $\lg N$ in worst case.
- A *min-heap* is basically the same, but with the minimum value at the root and children having larger values than their parents.

Example: Inserting into a simple heap

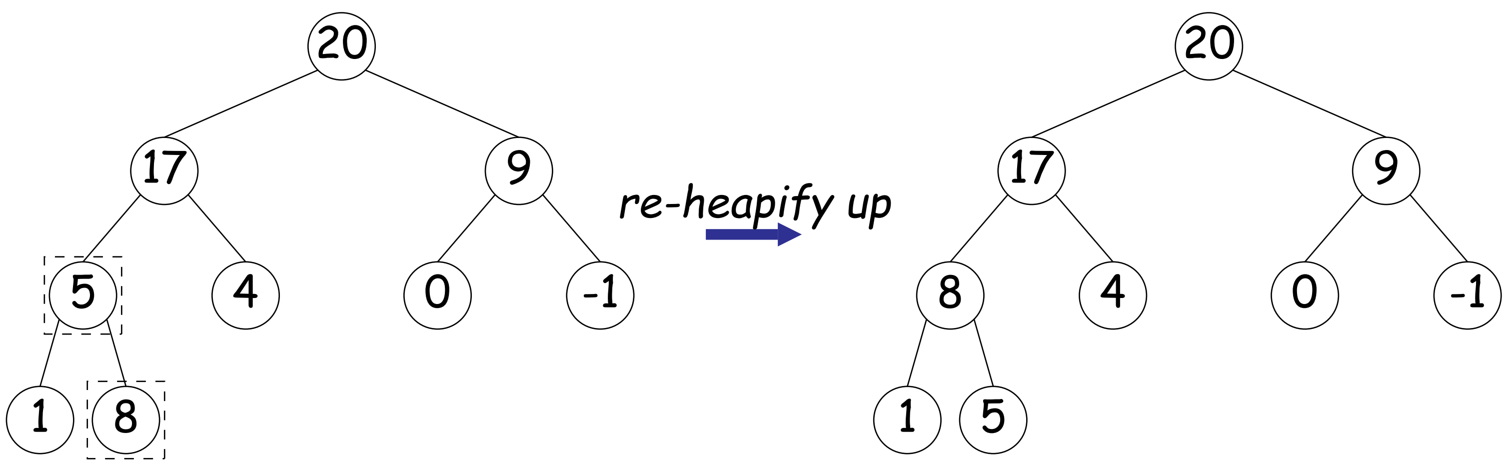
Data:

1 17 4 5 9 0 -1 20

Initial Heap:

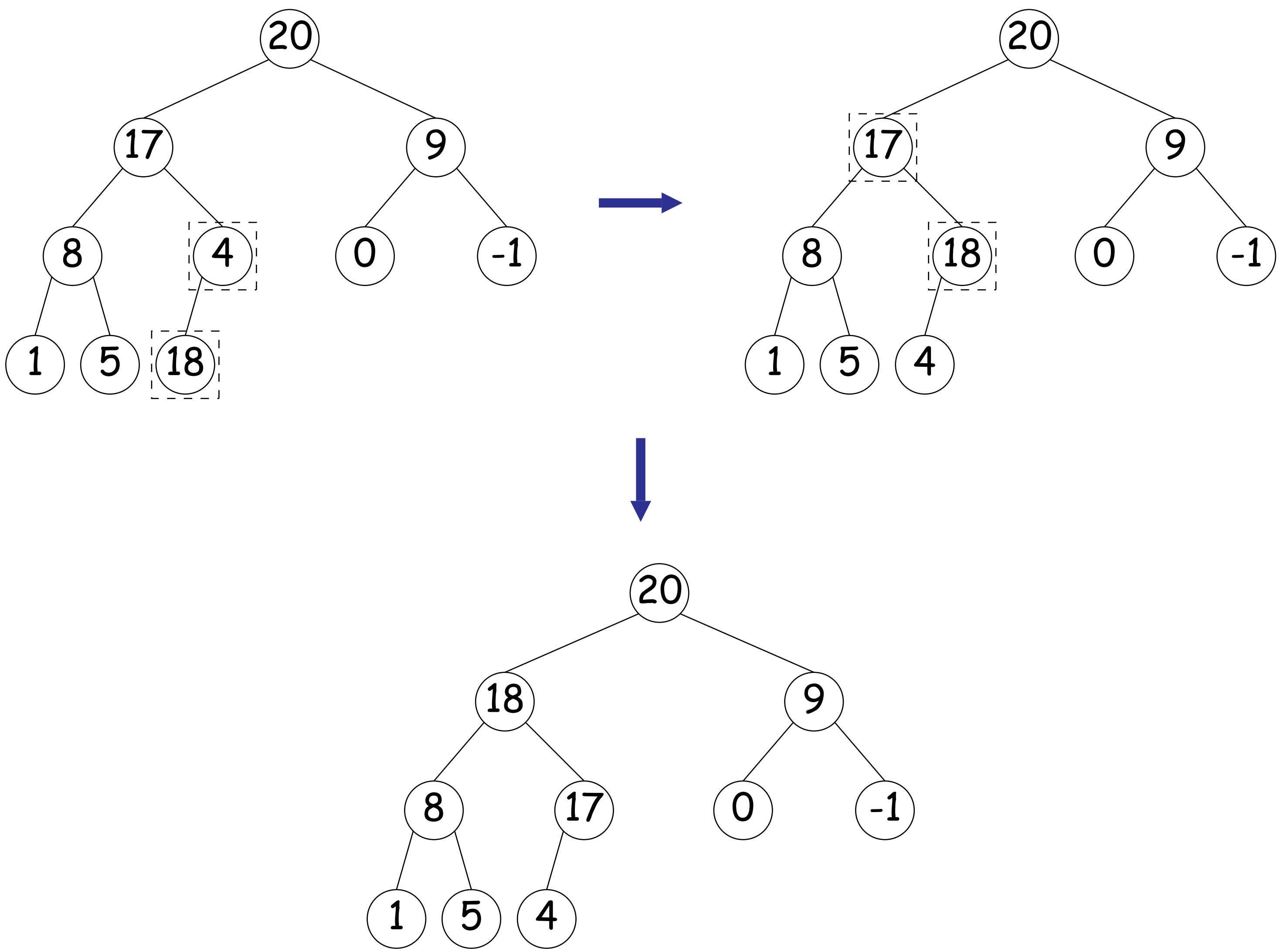


Add 8: Dashed boxes show where heap property violated



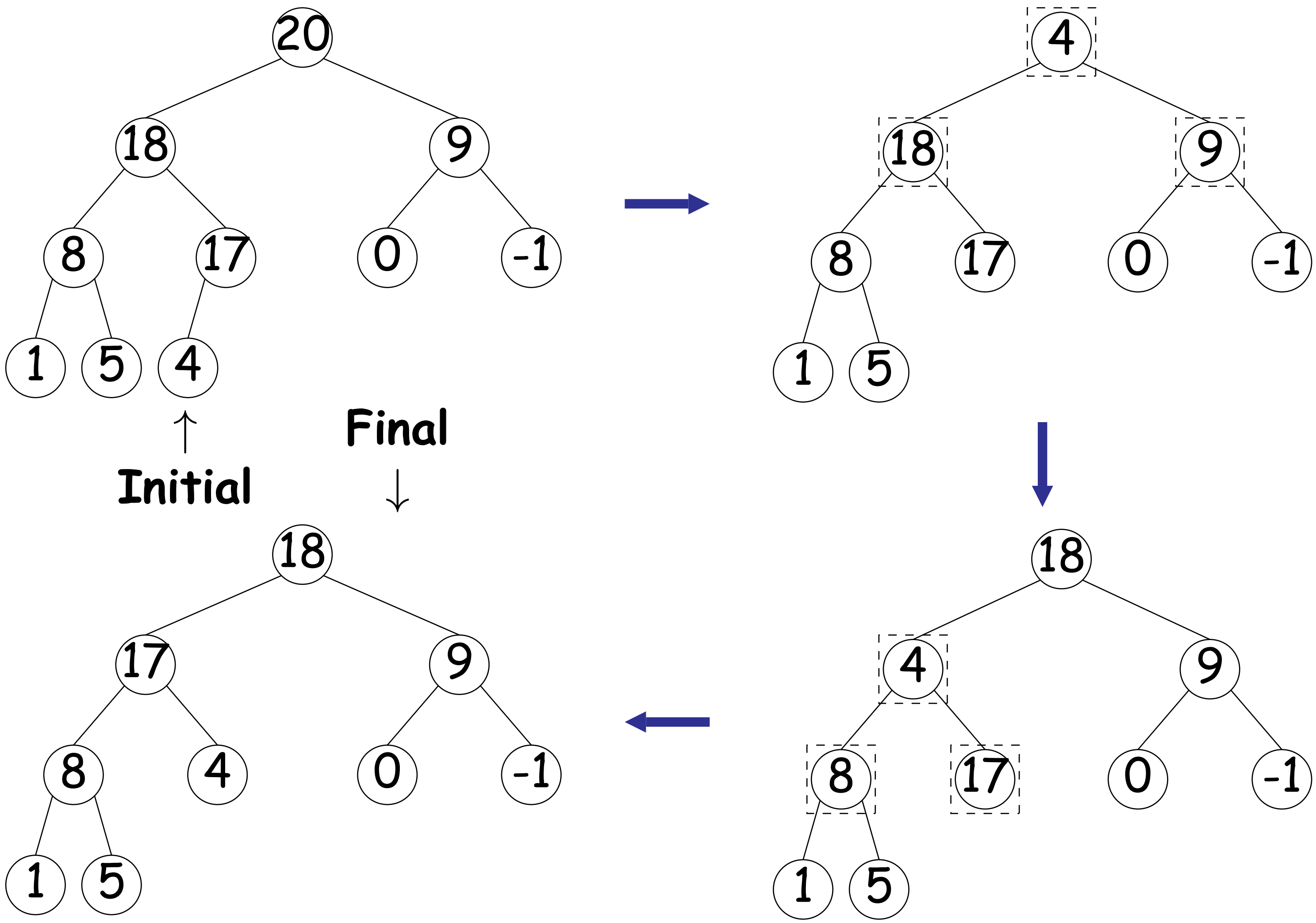
Heap insertion continued

Now insert 18:



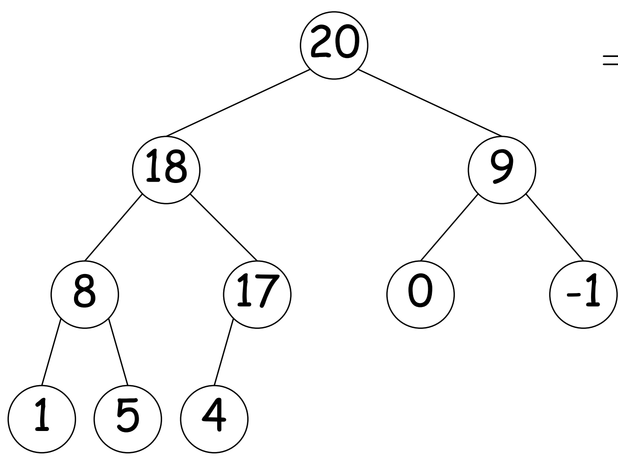
Removing Largest from Heap

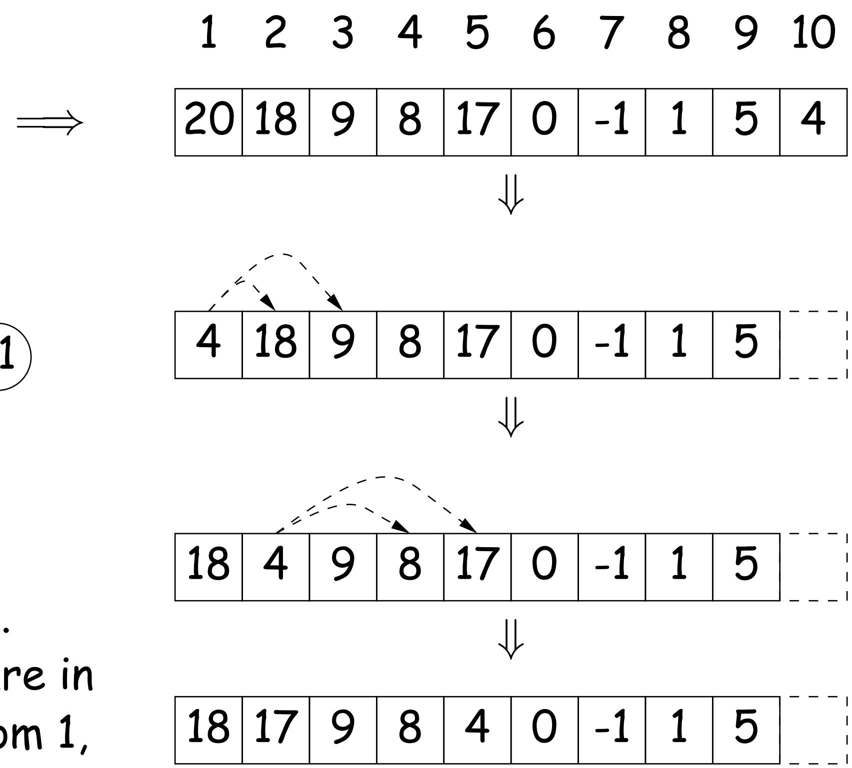
To remove largest: Move bottommost, rightmost node to top, then re-heapify down as needed (swap offending node with larger child) to re-establish heap property.



Heaps in Arrays

- Since heaps are nearly complete (missing items only at bottom level), can use arrays for compact representation.
- Example of removal from last slide (dashed arrows show children):





Nodes stored in level order.
Children of node at index $\#K$ are in $2K$ and $2K+1$ if numbering from 1, or $2K+1$ and $2K+2$ if from 0.

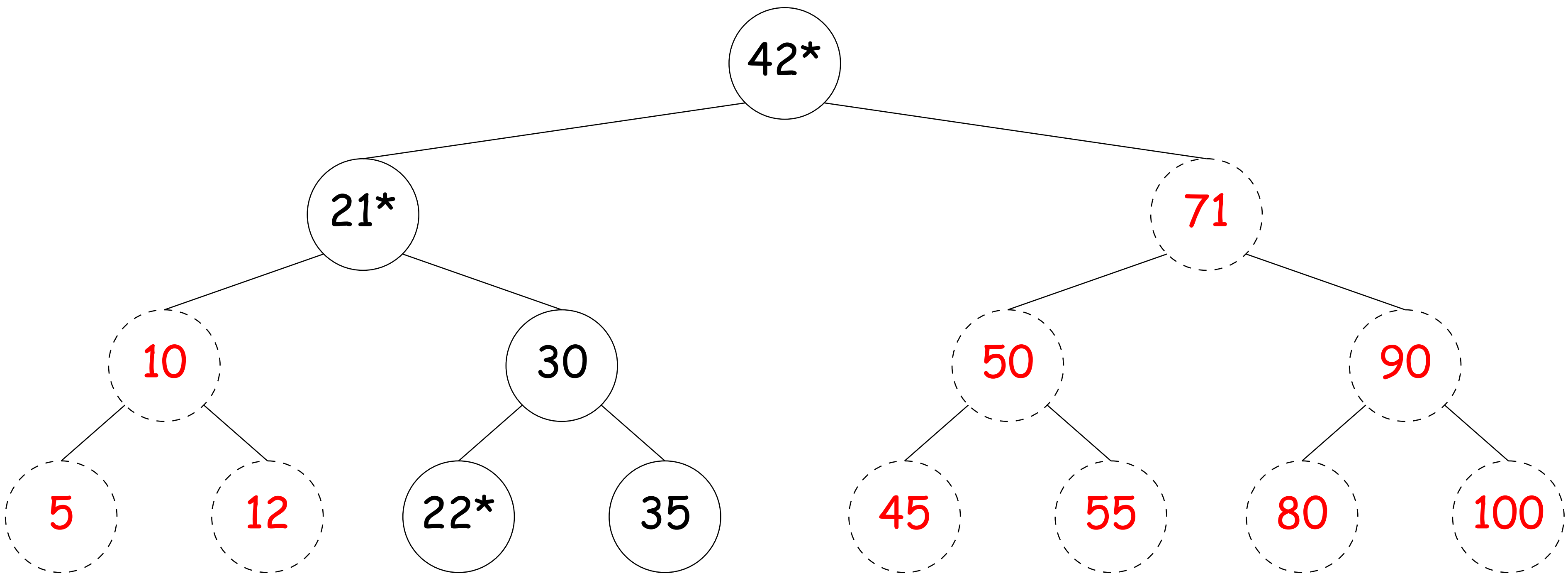
Ranges

- So far, have looked for specific items
- But for BSTs, need an ordering anyway, and can also support looking for *ranges of values*.
- Example: perform some action on all values in a BST that are within some range (in natural order):

```
/** Apply WHATTODO to all labels in T that are >= L and < U,
 *  in ascending natural order. */
static void visitRange(BST<String> T, String L, String U,
                       Consumer<BST<String>> whatToDo) {
  if (T != null) {
    int compLeft = L.compareTo(T.label ()),
        compRight = U.compareTo(T.label ());
    if (compLeft < 0)                    /* L < label */
        visitRange (T.left(), L, U, whatToDo);
    if (compLeft <= 0 && compRight > 0) /* L <= label < U */
        whatToDo.accept(T);
    if (compRight > 0)                   /* label < U */
        visitRange (T.right (), L, U, whatToDo);
  }
}
```

Time for Range Queries

- Time for range query $\in O(h+M)$, where h is height of tree, and M is number of data items that turn out to be in the range.
- Consider searching the tree below for all values $25 \leq x < 40$.
- Dashed nodes are never looked at. Starred nodes are looked at but not output. The h comes from the starred nodes; the M comes from unstarred non-dashed nodes.



Ordered Sets and Range Queries in Java

- Class `SortedSet` supports range queries with *views* of set:
  - `S.headSet(U)`: subset of `S` that is $<$ `U`.
  - `S.tailSet(L)`: subset that is $\geq$ `L`.
  - `S.subSet(L,U)`: subset that is $\geq$ `L`, $<$ `U`.
- Changes to views modify `S`.
- Attempts to, e.g., add to a `headSet` beyond `U` are disallowed.
- Can iterate through a view to process a range:

```
SortedSet<String> fauna = new TreeSet<String>
     (Arrays.asList ("axolotl", "elk", "dog", "hartebeest", "duck"));
for (String item : fauna.subSet ("bison", "gnu"))
    System.out.printf ("%s, ", item);
```

would print "`dog, duck, elk,`"

TreeSet

- Java library type `TreeSet<T>` requires either that `T` be `Comparable`, or that you provide a Comparator, as in:

```
SortedSet<String> rev_fauna = new TreeSet<String>(Collections.reverseOrder());
```

- Comparator is a type of function object:

```
interface Comparator<T> {
    /** Return <0 if LEFT<RIGHT, >0 if LEFT>RIGHT, else 0. */
    int compare(T left, T right);
}
```

(We'll deal with what `Comparator<T extends Comparable<T>>` is all about later.)

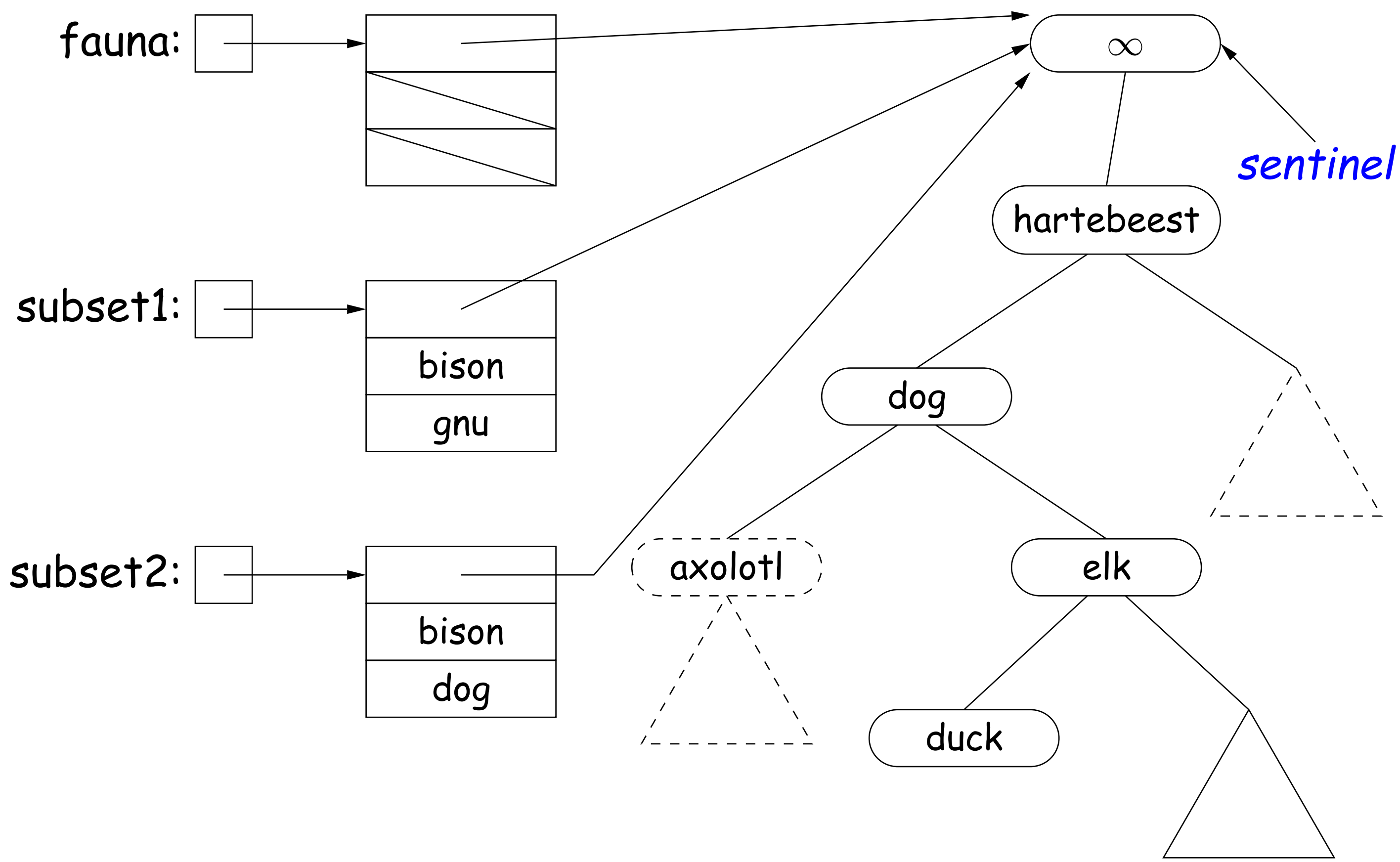
- For example, the `reverseOrder` comparator is defined like this:

```
/** A Comparator that gives the reverse of natural order. */
static <T extends Comparable<T>> Comparator<T> reverseOrder() {
   // Java figures out this lambda expression is a Comparator<T>.
   return (x, y) -> y.compareTo(x);
}
```

Example of Representation: BSTSet

- Same representation for both sets and subsets.
- Pointer to BST, plus bounds (if any).
- `.size()` is expensive!

```
SortedSet<String>
    fauna = new BSTSet<String>(stuff);
    subset1 = fauna.subSet("bison","gnu");
    subset2 = subset1.subSet("axolotl","dog");
```



Comparing Search Structures

Here, N is #items, k is #answers to query.

Function	Unordered List	Sorted Array	Bushy Search Tree	"Good" Hash Table	Heap
find	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(N)$
add (amortized)	$\Theta(1)$	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(\lg N)$
range query	$\Theta(N)$	$\Theta(k + \lg N)$	$\Theta(k + \lg N)$	$\Theta(N)$	$\Theta(N)$
find largest	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(1)$
remove largest	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(\lg N)$