CS61B Lectures #28

s on sorting by comparison

counting, radix sorts

ay: *DS(IJ)*, Chapter 8; Next topic: Chapter 9.

Better than N lg N?

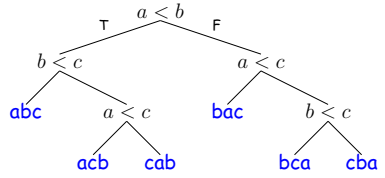
at *if all you can do to keys is compare them,* then sorting $N \lg N$).

there are $N!$ possible ways the input data could be

our program must be prepared to do $N!$ different combinations ng operations.

here must be $N!$ possible combinations of outcomes of sts in your program, since those determine what move where (we're assuming that comparisons are 2-way).

ree
ting time



Necessary Choices

if-test goes two ways, number of possible different k if-tests is 2^k.

nough tests so that $2^k \geq N!$, which means $k \geq \lg N!$.

g's approximation,

$$\begin{aligned} &\approx \sqrt{2\pi N}\left(\frac{N}{e}\right)^N \left(1 + \Theta\left(\frac{1}{N}\right)\right), \\ &\approx \frac{1}{2}(\lg 2\pi + \lg N) + N \lg N - N \lg e + \lg\left(1 + \Theta\left(\frac{1}{N}\right)\right) \\ &= \Theta(N \lg N) \end{aligned}$$

that k, the worst-case number of tests needed to sort omparison sorting, is in $\Omega(N \lg N)$: there must be cases ed (some multiple of) $N \lg N$ comparisons to sort N

Beyond Comparison: Distribution

we can do more than compare keys?

, how can we sort a set of N different integer keys range from 0 to kN, for some small constant k?

ue is *distribution sorting*:
tegers into N buckets; integer p goes to bucket $\lfloor p/k \rfloor$.
keys per bucket, so catenate and use insertion sort, now be fast.

$= 10$:

10 13 4 2 19 17 0 9

s:

2 | 4 | | 9 | 10 | 13 | 14 | 17 | 19 |

n sort is fast. Putting the data in buckets takes time sertion sort takes $\Theta(kN)$. When k is fixed (constant), ing in time $\Theta(N)$.

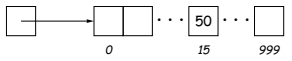
Distribution Counting

nique: *count* the number of items < 1, < 2, etc.

ems with value $< p$, then in sorted order, the j^{th} item must be item #$M_p + j$.

one has a set of numbers in the range $[0, 1000)$ (possibly es) and that exactly 15 of them are less than 50, which set. Then the result of sorting will look like this:

sult:



rds, the count of numbers $< k$ gives the index of k in rray.

N items in the range $0..M-1$, this gives another *linear-* $N)$)—algorithm (We include M and N here to allow for tes and for cases where $M \gg N$.)

ation: the notations $[A, B]$, (A, B), $[A, B)$, and $(A, B]$ above refer to *intervals*. theses vs. square brackets reflects the distinction between *open* and *closed* $[A, B]$ iff $A \leq x \leq B$, while $x \in [A, B)$ iff $A \leq x < B$, etc.]

Distribution Counting Example

tems are between 0 and 9 as in this example:

9	1	9	1	9	5	3	7	3	1	6	7	4	2	0

	2	2	1	1	3	0	3	*Counts*
	3	4	5	6	7	8	9	

	7	9	11	12	13	16	16	*Running sum*
2	< 3	< 4	< 5	< 6	< 7	< 8	< 9	

1	1	2	3	3	4	4	5	6	7	7	7	9	9	9
		6			9		11	12	13			16		

gives # occurrences of each key.

" gives cumulative count of keys $<$ each value...

s us where to put each key:

tance of key k goes into slot m, where m is the number ces that are $< k$; next k goes into slot $m+1$, etc.

istribution Counting Example (II)

9	1	9	1	9	5	3	7	3	1	6	7	4	2	0

	2	2	1	1	3	0	3	*Counts*
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3	4	5	6	7	8	9		

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									7						Output
		6			9			12			15			18	

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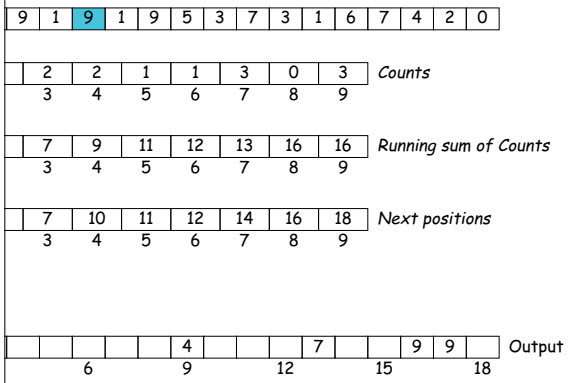

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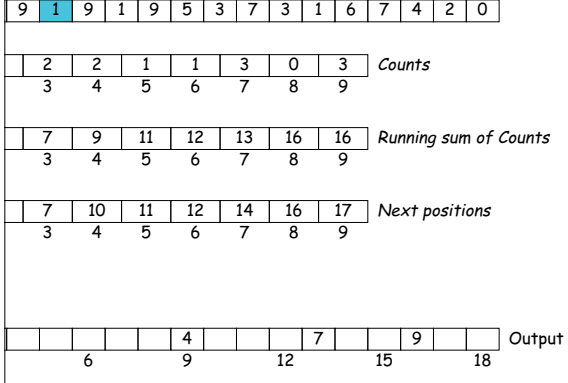

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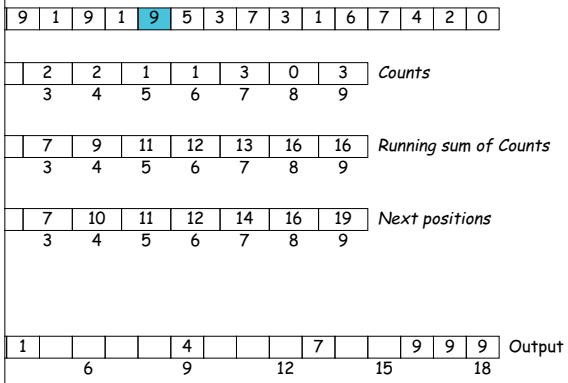

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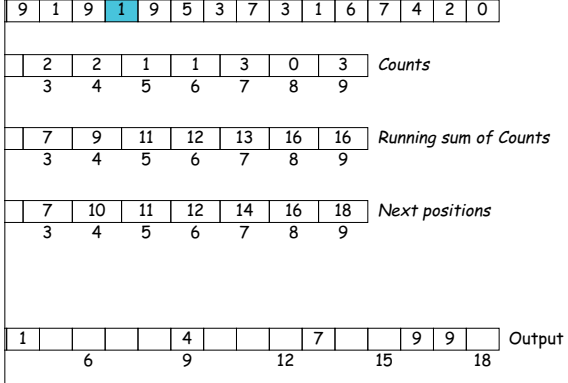

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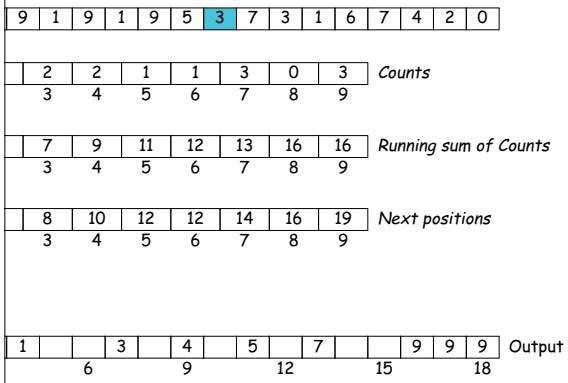

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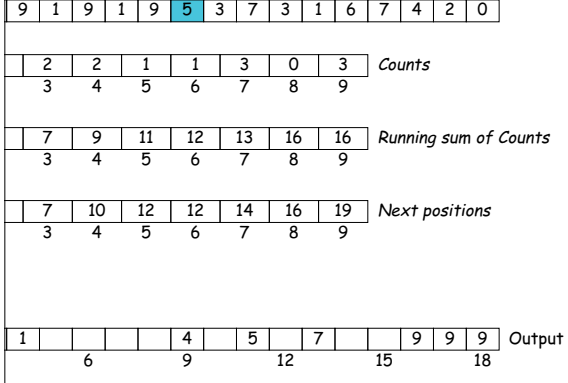

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Counts

	7	9	11	12	13	16	16
	3	4	5	6	7	8	9

Running sum of Counts

	9	10	12	12	15	16	19
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Next positions

1			3	3	4		5		7	7		9	9	9
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MSD Radix Sort

omplicated: must keep lists from each step separate

processing 1-element lists

A	posn
cat, cad, con, bat, can, be, let, bet	0
be, bet / cat, cad, con, can / let / set	1
$\star$ be, bet / cat, cad, con, can / let / set	2
be / bet / $\star$ cat, cad, con, can / let / set	1
be / bet / $\star$ cat, cad, can / con / let / set	2
be / bet / cad / can / cat / con / let / set	

s divide partially sorted sublists that will never be moved
e occupied by other sublists.

rk a sublist to be sorted on some character position.

Radix Sort

ys *one character at a time.*

ribution counting for each digit.

her right to left (LSD radix sort) or left to right (MSD

rt is venerable: used for punched cards. Example:

Initial: set, cat, cad, con, bat, can, be, let, bet

bet
let
bat
can cat
e cad con set
' 'd' 'n' 't'

n, can, set, cat, bat, let, bet

Pass 2 (by char #1)

bat bet
cat let
can set
cad be con
'a' 'e' 'o'

cad, can, cat, bat, be, set, let, bet, con

Pass 3 (by char #0)

con
bet cat
be can
bat cad let set
'b' 'c' 'l' 's'

bat, be, bet, cad, can, cat, con, let, set

And Don't Forget Search Trees

h tree is in sorted order, when read in inorder.

e to really use for sorting [next topic].

e, same performance as heapsort: N insertions in time
us $\Theta(N)$ to traverse, gives

$$\Theta(N + N \lg N) = \Theta(N \lg N)$$

Performance of Radix Sort

akes $\Theta(B)$ time where B is *total size of the key data*.

ed other sort times as functions of #records.

are?

ifferent records, one must have keys at least $\Theta(\lg N)$

, comparison actually takes time $\Theta(K)$ where K is size
st case [why?]

mparisons really means $N(\lg N)^2$ operations.

sort would take $B = N \lg N$ time for N records with
th ($\lg N$) keys.

hand, we must work to get good constant factors with

Summary

rt: $\Theta(Nk)$ comparisons and moves, where k is maximum
is displaced from final position.

small datasets or almost ordered data sets.

$(N \lg N)$ with good constant factor if data is not pathological.
$O(N^2)$.

$\Theta(N \lg N)$ guaranteed. Good for external sorting.

eesort with guaranteed balance: $\Theta(N \lg N)$ guaranteed.

istribution sort: $\Theta(B)$ (number of bytes). Also good for
ting.