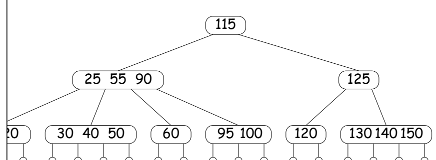
CS61B Lecture #29

Balanced Search: The Problem

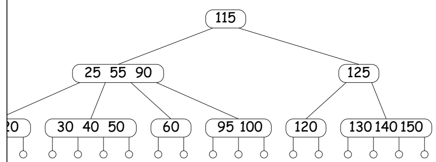
- rch trees important?
- /deletion fast (on every operation, unlike hash table, to expand from time to time).
- ange queries, sorting (unlike hash tables)
- performance from binary search tree requires remaining led ≈ by some some constant > 1 at each node.
- ds, that tree be "bushy"
- es (most inner nodes with one child) perform like linked
- hat heights of any two subtrees of a node always differ no more than some constant factor $C > 0$.

mple of Direct Approach: B-Trees



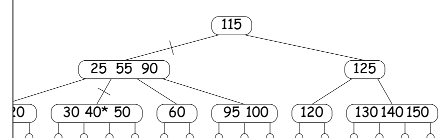
- tree is an M-ary search tree, $M > 2$.
- , if $M = 4$, non-root nodes have at least 2 nodes, so we e (2, 4) (or 2-4) trees.
- earch-tree property:
  - sorted in each node.
  - subtrees to the left of a key, K, are $< K$, and all to are $> K$.
  - he bottom of tree are all empty (don't really exist) and rom root.
  - a simple generalization of binary search.

mple of Direct Approach: B-Trees



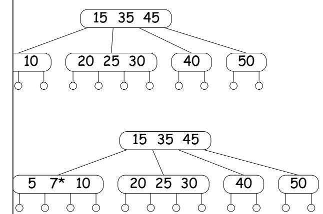
- ree grows/shrinks only at the root, then the two sides e height.
- xcept the root, has from $\lceil M/2 \rceil$ to M children, and one " each two children.
- from 2 to M children (in a non-empty tree).
- d to nodes just above the (empty) leaves; split overfull ded, moving one key up to its parent.

mple Order 4 B-tree ((2,4) Tree)

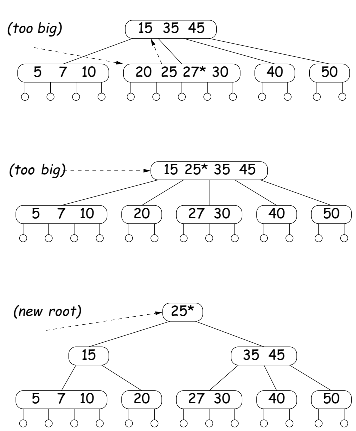


- s show path when finding 40.
- er side of each child pointer in path bracket 40.
- as at least 2 children, and all leaves (little circles) are m, so height must be $O(\lg N)$.
- B-trees are stored on secondary storage, and the order uch bigger
- le to the size of a disk sector, page, or other convenient O.

erting in (2, 4) tree (Simple Case)

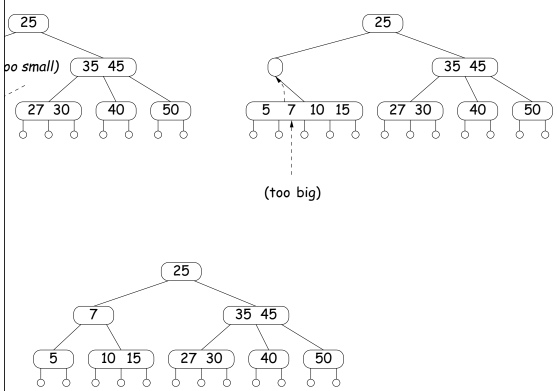


Inserting in B-Tree (Splitting)



Deleting Keys from B-tree

rom last tree.



Red-Black Trees

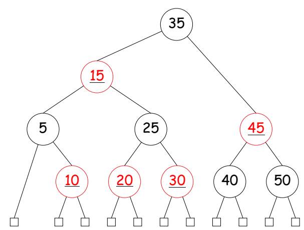
tree is a binary search tree with additional constraints
w unbalanced it can be.

ing is always $O(\lg N)$.

a's `TreeSet` and `TreeMap` types.

are inserted or deleted, the tree is *rotated* and *recolored*
restore balance.

Red-Black Tree Constraints



(conceptually) colored red or black.

.

ode contains no data (as for B-trees) and is black.

as same number of black ancestors.

al node has two children.

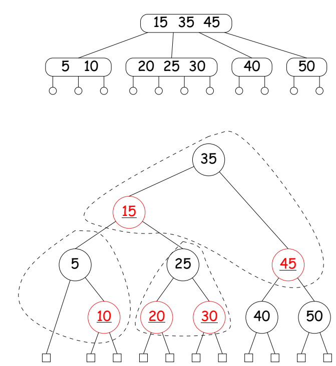
de has two black children.

, 5, and 6 guarantee $O(\lg N)$ searches.

ed-Black Trees and (2,4) Trees

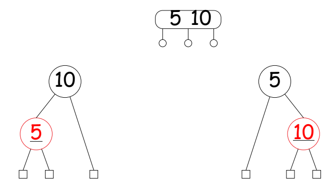
ack tree corresponds to a (2,4) tree, and the operations
spond to those on the other.

f (2,4) tree corresponds to a cluster of 1–3 red-black
h the top node is black and any others are red.



onstraints: Left-Leaning Red-Black Trees

,4) or (2,3) tree with three children may be represented
ent ways in a red-black tree:



iderably simplify insertion and deletion in a red-black
ys choosing the option on the left.

onstraint, there is a one-to-one relationship between
nd red-black trees.

g trees are called *left-leaning red-black trees.*

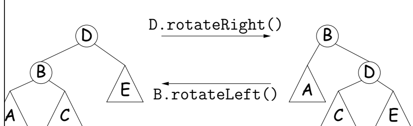
r simplification, let's restrict ourselves to red-black
correspond to (2,3) trees (whose nodes have no more
en), so that no red-black node has two red children.

ed-Black Insertion and Rotations

ttom just as for binary tree (color red except when tree
y).

(and recolor) to restore red-black property, and thus

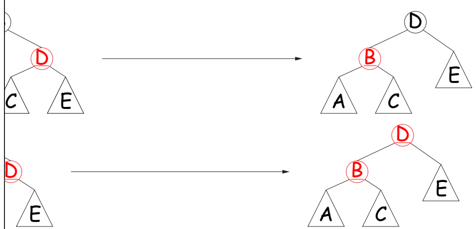
rees *preserves* binary tree property, but changes balance.



Rotations and Recolorings

oses, we'll augment the general rotation algorithms with
ing.

color from the original root to the new root, and color
root red. Examples:

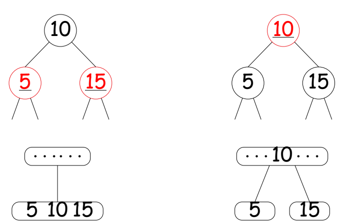


hese changes the number of black nodes along any path
root and the leaves.

Splitting by Recoloring

ns will temporarily create nodes with too many children,
t them up.

oloring allows us to split nodes. We'll call it `colorFlip`:



joins the parent node, splitting the original.

The Algorithm (Sedgewick)

binary-tree type `RBTree`: basically ordinary BST nodes

the same as for ordinary BSTs, but we add some fixups
he red-black properties.

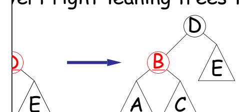
```
rt(RBTree tree, KeyType key) {
e == null)
urn new RBTree(key, null, null, RED);
 = key.compareTo(tree.label());
 (cmp < 0) tree.setLeft(insert(tree.left(), key));
          tree.setRight(insert(tree.right(), key));

fixup(tree);     // Only line that's all new!
```


Fixing Up the Tree

back up the BST, we restore the left-leaning red-black
nd limit ourselves to red-black trees that correspond
s by applying the following (in order) to each node:

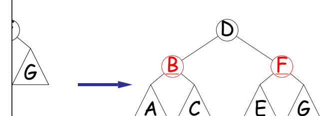
ert right-leaning trees to left-leaning:



```
if (tree.right().isRed()
    && tree.left().isBlack()) {
      tree.rotateLeft();
}
```

node B will be red, so that both B and D end up red. This
.

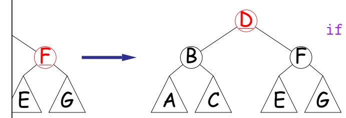
ate linked red nodes into a normal 4-node (temporarily).



```
if (tree.left().isRed() &&
    tree.left().left().isRed())
      tree.rotateRight();
```


Fixing Up the Tree (II)

ak up 4-nodes into 3-nodes or 2-nodes.

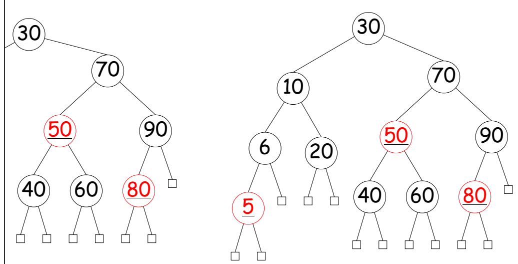


```
if (tree.left().isRed() &&
    tree.right().isRed())
      colorFlip(tree);
```

a result of other fixups, or of insertion into the empty
t may end up red, so color the root black after the rest
nd fixups are finished. (Not part of the fixup function;
the end).

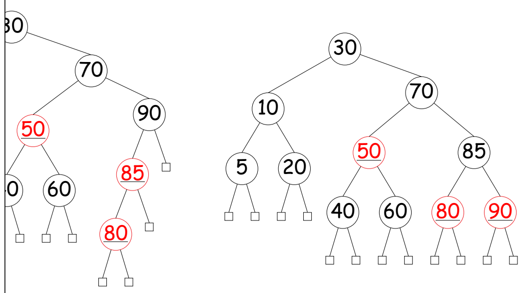
Insertion Example (II)

), let's insert 6, leading to the tree on the left. This is
, so apply Fixup 1:



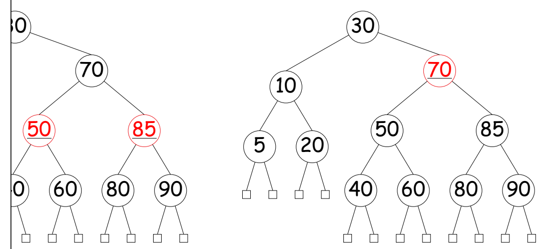
Insertion Example (IIIa)

xup 2.



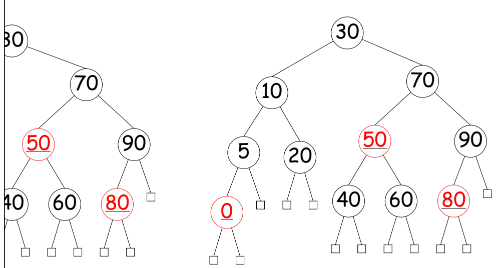
Insertion Example (IIIc)

another 4-node, so apply fixup 3 again.



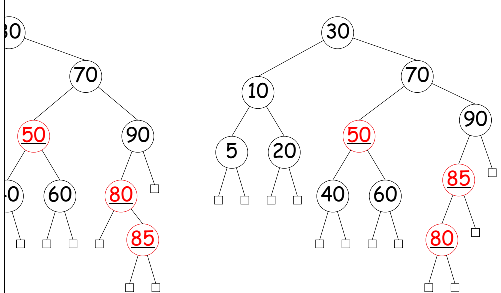
of Left-Leaning 2-3 Red-Black Insertion

initial tree on left. No fixups needed.



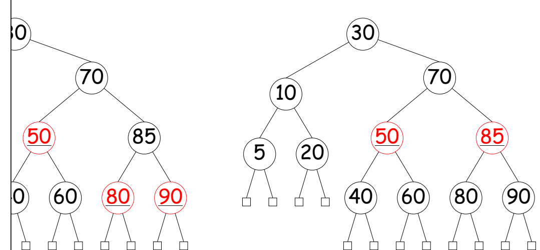
Insertion Example (III)

r inserting 85. We need fixup 1 first.



Insertion Example (IIIb)

a 4-node, so apply fixup 3.



Insertion Example (IIId)

a right-leaning tree, so apply fixup 1.