CS61B Lecture #31

Today:

• More balanced search structures (DS(IJ), Chapter 9
Really Efficient Use of Keys: the Trie

- Haven’t said much about cost of comparisons.
- For strings, worst case is length of string.
- Therefore should throw extra factor of key length, $L$, into costs:
  - $\Theta(M)$ comparisons really means $\Theta(ML)$ operations.
  - So to look for key $X$, keep looking at same chars of $X$ $M$ times.
- Can we do better? Can we get search cost to be $O(L)$?

Idea: Make a multi-way decision tree, with one decision per character of key.
The Trie: Example

- Set of keys
  \{a, abase, abash, abate, abbas, axolotl, axe, fabric, facet\}
- Ticked lines show paths followed for “abash” and “fabric”
- Each internal node corresponds to a possible prefix.
- Characters in path to node = that prefix.
Adding Item to a Trie

- Result of adding bat and faceplate.
- New edges ticked.
A Side-Trip: Scrunching

- For speed, obvious implementation for internal nodes is array indexed by character.

- Gives $O(L)$ performance, $L$ length of search key.

- [Looks as if independent of $N$, number of keys. Is there a dependence?]?

- Problem: arrays are *sparsely populated* by non-null values—waste of space.

**Idea:** Put the arrays on top of each other!

- Use null (0, empty) entries of one array to hold non-null elements of another.

- Use extra markers to tell which entries belong to which array.
Scrunching Example

Small example: (unrelated to Tries on preceding slides)

- Three arrays, each indexed 0..9

A1:

```
0 1 2 3 4 5 6 7 8 9
bass trout pike
```

A2:

```
0 1 2 3 4 5 6 7 8 9
ghee milk oil
```

A3:

```
0 1 2 3 4 5 6 7 8 9
salt cumin mace
```

- Now overlay them, but keep track of the original index of each item:

Check:

```
0 -1 1 -1 2 5 5 7 6 7 9
```

A123:

```
0* 1 2 3 4 5* 6 7* 8 9
```

A3: 0 1* 2 3 4 5* 6 7 8 9*  
A2: 0 1 2* 3 4 5 6* 7* 8 9  
A1: 0* 1 2 3 4 5* 6 7* 8 9

Starred items are null in uncompressed array

Index in original array or -1 if null in all arrays
Scrunching Example (contd.)

```c
/* A2[i] == */ (Check[i + 2] == i) ? A123[i + 2] : null;
/* A3[i] == */ (Check[i + 1] == i) ? A123[i + 1] : null;
```
Practicum

• The scrunching idea is cute, but
  - Not so good if we want to expand our trie.
  - A bit complicated.
  - Actually more useful for representing large, sparse, fixed tables with many rows and columns.

• Furthermore, number of children in trie tends to drop drastically when one gets a few levels down from the root.

• So in practice, might as well use linked lists to represent set of node’s children...

• ...but use arrays for the first few levels, which are likely to have more children.
Probabilistic Balancing: Skip Lists

- A skip list can be thought of as a kind of n-ary search tree in which we choose to put the keys at “random” heights.

- More often thought of as an ordered list in which one can skip large segments.

- Typical example:

- To search, start at top layer on left, search until next step would overshoot, then go down one layer and repeat.

- In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.

- Heights of the nodes were chosen randomly so that there are about $1/2$ as many nodes that are $> k$ high as there are that are $k$ high.

- Makes searches fast with high probability.
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  ![Skip List Example](image)

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- Typical example:

```
<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>
```

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- Typical example:

```
\[
\begin{array}{cccccccccccc}
\infty & 10 & 20 & 25 & 30 & 40 & 50 & 55 & 60 & 90 & 95 & 100 & 115 & 120 & 125 & 130 & 140 & 150 & \infty
\end{array}
\]
```

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<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>10</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>90</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>100</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>120</td>
<td>125</td>
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<tr>
<td></td>
<td>55</td>
<td>130</td>
<td>130</td>
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- In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.

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\downarrow \\
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25 \\
30 \\
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55 \\
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95 \\
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140 \\
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\infty
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Example: Adding and deleting

- Starting from initial list:

- In any order, we add 126 and 127 (choosing random heights for them), and remove 20 and 40:

- Shaded nodes here have been modified.
Summary

• Balance in search trees allows us to realize $\Theta(lg N)$ performance.

• B-trees, red-black trees:
  - Give $\Theta(lg N)$ performance for searches, insertions, deletions.
  - B-trees good for external storage. Large nodes minimize # of I/O operations

• Tries:
  - Give $\Theta(B)$ performance for searches, insertions, and deletions, where $B$ is length of key being processed.
  - But hard to manage space efficiently.

• Interesting idea: scrunched arrays share space.

• Skip lists:
  - Give probable $\Theta(lg N)$ performance for searches, insertions, deletions
  - Easy to implement.
  - Presented for interesting ideas: probabilistic balance, randomized data structures.
Summary of Collection Abstractions

- **Multiset**
  - contains, iterator
- **List**
  - get(n)
- **Set**
- **Ordered Set**
  - first
  - subset
- **Unordered Set**
- **Priority Queue**
- **Sorted Set**
- **Map**
  - contains, iterator
  - get
- **Unordered Map**
- **Ordered Map**

Blue: Java has corresponding interface

Green: Java has no corresponding interface
Data Structures that Implement Abstractions

Multiset

- List: arrays, linked lists, circular buffers
- Set
  - OrderedSet
    * Priority Queue: heaps
    * Sorted Set: binary search trees, red-black trees, B-trees, sorted arrays or linked lists
  - Unordered Set: hash table

Map

- Unordered Map: hash table
- Ordered Map: red-black trees, B-trees, sorted arrays or linked lists
Corresponding Classes in Java

**Multiset** (Collection)

- **List**: ArrayList, LinkedList, Stack, ArrayBlockingQueue, ArrayDeque

- **Set**
  - **OrderedSet**
    - **Priority Queue**: PriorityQueue
    - **Sorted Set** (SortedSet): TreeSet
  - **Unordered Set**: HashSet

**Map**

- **Unordered Map**: HashMap

- **Ordered Map** (SortedMap): TreeMap