

## Dynamic Programming

Garcia):

on a list with an even number of non-negative integers. Each player in turn takes either the leftmost number or the rightmost number. You get the largest possible sum.

For example, with (6, 12, 0, 8), you (as first player) should take 12, and when the second player takes, you also get the 12, for a total of 24.

Can you write a program that, given a list of numbers, determines whether your opponent plays perfectly (i.e., to get as much as possible), and if so, how to maximize your sum?

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## Still Another Idea from CS61A

is that we are recomputing intermediate results many times.

memoize the intermediate results. Here, we pass in an array of memoized results, initialized to -1.

```
bestSum(int[] V, int left, int right, int total, int[][] memo) {
    if (left > right) {
        return 0;
    }
    if (memo[left][right] == -1) {
        int L = total - bestSum(V, left+1, right, total-V[left], memo);
        int R = total - bestSum(V, left, right-1, total-V[right], memo);
        memo[left][right] = Math.max(L, R);
    }
    return memo[left][right];
}
```

The number of recursive calls to bestSum must be  $O(N^2)$ , for a list of length  $N$ , an enormous improvement from  $\Theta(2^N)$ !

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## Longest Common Subsequence

length of the longest string that is a subsequence of both strings.

Longest common subsequence of "111sea shells by the seashore" and "1d salt sellers at the salt mines"

Longest common subsequence of "111sea shells by the seashore" (length 23)

Longest common subsequence of "1d salt sellers at the salt mines" (length 13)

Longest common subsequence of "111sea shells by the seashore" and "1d salt sellers at the salt mines" (length 13)

```
longestCommonSubsequence(String S0, String S1, int k0, int k1) {
    if (k0 == 0 || k1 == 0) return 0;
    if (S0.charAt(k0-1) == S1.charAt(k1-1)) return 1 + longestCommonSubsequence(S0, k0-1, S1, k1-1);
    return Math.max(longestCommonSubsequence(S0, k0-1, S1, k1), longestCommonSubsequence(S0, k0, S1, k1-1));
}
```

but obviously memoizable.

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## CS61B Lecture #36

Dynamic Programming

Trip: Enumeration types.

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## Obvious Program

Makes it easy, again:

```
bestSum(int[] V) {
    int total = 0;
    for (int i = 0; i < V.length; i++) total += V[i];
    return bestSum(V, 0, V.length-1, total);
}

longestSum(int[] V, int left, int right, int total) {
    if (left > right) {
        return 0;
    }
    int L = total - bestSum(V, left+1, right, total-V[left]);
    int R = total - bestSum(V, left, right-1, total-V[right]);
    return Math.max(L, R);
}
```

$C(0) = 1$ ,  $C(N) = 2C(N-1)$ ; so  $C(N) \in \Theta(2^N)$

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## Iterative Version

Like the recursive version, but the usual presentation of this is as *dynamic programming*—is iterative:

```
longestCommonSubsequence(String S0, String S1) {
    int[] memo = new int[S0.length()+1][S1.length()+1];
    for (int i = 0; i < S0.length(); i++) {
        for (int j = 0; j < S1.length(); j++) {
            if (S0.charAt(i) == S1.charAt(j)) {
                memo[i+1][j+1] = memo[i][j] + 1;
            } else {
                memo[i+1][j+1] = Math.max(memo[i+1][j], memo[i][j+1]);
            }
        }
    }
    return memo[S0.length()+1][S1.length()+1];
}
```

Figure out ahead of time the order in which the memoized results should be computed, and write an explicit loop, saving the time to check whether result exists.

Why bother unless it's necessary to save space?

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## Memoized Longest Common Subsequence

```
longest common subsequence of S0[0..k0-1] and S1[0..k1-1]. */
String S0, int k0, String S1, int k1) {
    = new int[k0+1][k1+1];
    bw : memo) Arrays.fill(row, -1);
    0, k0, S1, k1, memo);

int lls(String S0, int k0, String S1, int k1, int[][] memo) {
    || k1 == 0) return 0;
    [k1] == -1) {
    0-1] == S1[k1-1])
    [k0][k1] = 1 + lls(S0, k0-1, S1, k1-1, memo);

[k0][k1] = Math.max(lls(S0, k0-1, S1, k1, memo),
                    lls(S0, k0, S1, k1-1, memo));

k0][k1];
```

Will the memoized version be?  $\Theta(k_0 \cdot k_1)$

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## Enum Types in Java

enum versions of Java allow syntax like that of C or C++, but guarantees:

```
enum Piece
    PIECE, BLACK_KING, WHITE_PIECE, WHITE_KING, EMPTY; }

enum as a new reference type, a special kind of class type.
BLACK_PIECE, etc., are static, final enumeration constants
of type PIECE.
```

enum are automatically initialized, and are the only values of the type that exist (cannot say new Piece()).

enum can be used in ==, and also switch statements:

```
public boolean isKing(Piece p) {
    } {
    KING: case WHITE_KING: return true;
    return false;
}
```

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## Operations on Enum Types

enum Declaration of enumeration constants significant: ordinal() method (numbering from 0) of an enumeration value. Thus, BLACK\_KING.ordinal() is 1.

enum Piece.values() gives all the possible values of the type. enum can be written:

```
enum Piece {
    ...
};
System.out.println(Piece.values());
System.out.printf("Piece value %d is %s\n", p.ordinal(), p);
```

enum Piece.valueOf() converts a String into a value of the type. So Piece.valueOf("EMPTY") == EMPTY.

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## Memoized Longest Common Subsequence

```
longest common subsequence of S0[0..k0-1] and S1[0..k1-1]. */
String S0, int k0, String S1, int k1) {
    = new int[k0+1][k1+1];
    bw : memo) Arrays.fill(row, -1);
    0, k0, S1, k1, memo);

int lls(String S0, int k0, String S1, int k1, int[][] memo) {
    || k1 == 0) return 0;
    [k1] == -1) {
    0-1] == S1[k1-1])
    [k0][k1] = 1 + lls(S0, k0-1, S1, k1-1, memo);

[k0][k1] = Math.max(lls(S0, k0-1, S1, k1, memo),
                    lls(S0, k0, S1, k1-1, memo));

k0][k1];
```

Will the memoized version be?

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## Trip into Java: Enumeration Types

enum is a type with a few, named, discrete values.

enum In most forms, the only necessary operations are == and !=; the only property of a value of the type is that it differs from all other values.

enum In versions of Java, used named integer constants:

```
enum Pieces {
    NONE = 0, // Fields in interfaces are static final.
    KING = 1,
    QUEEN = 2,
    ROOK = 3,
    PAWN = 4;
}
```

enum can be used as a shorthand, with syntax like

```
enum Pieces {
    NONE, KING, QUEEN, ROOK, PAWN;
}
```

enum In these values are basically ints, accidents can happen.

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## Using Enumerations Available Elsewhere

enum In Java, enum values like BLACK\_PIECE are static members of a class, not classes.

enum In Java, unlike C or C++, their declarations are not automatically associated with the enumeration class definition.

enum In Java, to use enum values in other classes, must write Piece.BLACK\_PIECE, which can get messy.

enum In Java, with version 1.5, Java has *static imports*: to import all members of a class checker.Piece (including enumerations), you can write

```
import static checker.Piece.*;
```

enum In Java, you can use import clauses.

enum In Java, you can use this for enum classes in the anonymous package.

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## Fancy Enum Types

esses. You can define all the extra fields, methods, and ; you want.

s are used only in creating enumeration constants. The arguments follow the constant name:

```
CE(BLACK, false, "b"), BLACK_KING(BLACK, true, "B"),
CE(WHITE, false, "w"), WHITE_KING(WHITE, true, "W"),
l, false, " ");

final Side color;
final boolean isKing;
final String textName;

e color, boolean isKing, String textName) {
lor = color; this.isKing = isKing; this.textName = textName;

r() { return color; }
sKing() { return isKing; }
xtName() { return textName; }
```