

CS61B Lecture #36

Today:

- Dynamic Programming
- A Brief Side Trip: Enumeration types.

Dynamic Programming

- A puzzle (D. Garcia):
 - Start with a list with an even number of non-negative integers.
 - Each player in turn takes either the leftmost number or the rightmost.
 - Idea is to get the largest possible sum.
- Example: starting with (6, 12, 0, 8), you (as first player) should take the 8. Whatever the second player takes, you also get the 12, for a total of 20.
- Assuming your opponent plays perfectly (i.e., to get as much as possible), how can you maximize your sum?
- Can solve this with exhaustive game-tree search, but...

Obvious Program

- Recursion makes it easy, again:

```
int bestSum(int[] V) {
    int total, i, N = V.length;
    for (i = 0, total = 0; i < N; i += 1) total += V[i];
    return bestSum(V, 0, N-1, total);
}

/** The largest sum obtainable by the first player in the game on
 * the list V[LEFT..RIGHT], given TOTAL = sum of V[LEFT..RIGHT]. */
int bestSum(int[] V, int left, int right, int total) {
    if (left > right) {
        return 0;
    } else {
        int L = total - bestSum(V, left+1, right, total-V[left]);
        int R = total - bestSum(V, left, right-1, total-V[right]);
        return Math.max(L, R);
    }
}
```

- Time cost is $C(0) = 1$, $C(N) = 2C(N - 1)$; so $C(N) \in \Theta(2^N)$

Still Another Idea from CS61A

- The problem is that we are recomputing intermediate results many times.
- Solution: *memoize* the intermediate results. Here, we pass in an $N \times N$ array ($N = V.length$) of memoized results, initialized to -1.

```
int bestSum(int[] V, int left, int right, int total, int[][] memo) {
    if (left > right) {
        return 0;
    } else if (memo[left][right] == -1) {
        int L = total - bestSum(V, left+1, right, total-V[left], memo);
        int R = total - bestSum(V, left, right-1, total-V[right], memo);
        memo[left][right] = Math.max(L, R);
    }
    return memo[left][right];
}
```

- Now the number of recursive calls to `bestSum` must be $O(N^2)$, for $N =$ the length of V , an enormous improvement from $\Theta(2^N)$!

Iterative Version

- I prefer the recursive version, but the usual presentation of this idea—known as *dynamic programming*—is iterative:

```
int bestSum(int[] V) {
    int[][] memo = new int[V.length][V.length];
    int[][] total = new int[V.length][V.length];
    for (int i = 0; i < V.length; i += 1)
        memo[i][i] = total[i][i] = V[i];
    for (int k = 1; k < V.length; k += 1)
        for (int i = 0; i < V.length-k-1; i += 1) {
            total[i][i+k] = V[i] + total[i+1][i+k];
            int L = total[i][i+k] - memo[i+1][i+k];
            int R = total[i][i+k] - memo[i][i+k-1];
            memo[i][i+k] = Math.max(L, R);
        }
    return memo[0][V.length-1];
}
```

- That is, we figure out ahead of time the order in which the memoized version will fill in `memo`, and write an explicit loop, saving the time needed to check whether result exists.
- But I say, why bother unless it's necessary to save space?

Longest Common Subsequence

- **Problem:** Find length of the longest string that is a subsequence of each of two other strings.
- **Example:** Longest common subsequence of "sally_sells_sea_shells_by_the_seashore" and "sarah_sold_salt_sellers_at_the_salt_mines" is "sa_sl_sa_sells_the_sae" (length 23)
- Similarity testing, for example.
- Obvious recursive algorithm:

```
/** Length of longest common subsequence of S0[0..k0-1]
 * and S1[0..k1-1] (pseudo Java) */
static int lls(String S0, int k0, String S1, int k1) {
    if (k0 == 0 || k1 == 0) return 0;
    if (S0[k0-1] == S1[k1-1]) return 1 + lls(S0, k0-1, S1, k1-1);
    else return Math.max(lls(S0, k0-1, S1, k1), lls(S0, k0, S1, k1-1));
}
```

- Exponential, but obviously memoizable.

Memoized Longest Common Subsequence

```
/** Length of longest common subsequence of S0[0..k0-1] and S1[0..k1-1]. */
static int lls(String S0, int k0, String S1, int k1) {
    int[][] memo = new int[k0+1][k1+1];
    for (int[] row : memo) Arrays.fill(row, -1);
    return lls(S0, k0, S1, k1, memo);
}
private static int lls(String S0, int k0, String S1, int k1, int[][] memo) {
    if (k0 == 0 || k1 == 0) return 0;
    if (memo[k0][k1] == -1) {
        if (S0[k0-1] == S1[k1-1])
            memo[k0][k1] = 1 + lls(S0, k0-1, S1, k1-1, memo);
        else
            memo[k0][k1] = Math.max(lls(S0, k0-1, S1, k1, memo),
                                    lls(S0, k0, S1, k1-1, memo));
    }
    return memo[k0][k1];
}
```

Q: How fast will the memoized version be?

Memoized Longest Common Subsequence

```
/** Length of longest common subsequence of S0[0..k0-1] and S1[0..k1-1]. */
static int lls(String S0, int k0, String S1, int k1) {
    int[][] memo = new int[k0+1][k1+1];
    for (int[] row : memo) Arrays.fill(row, -1);
    return lls(S0, k0, S1, k1, memo);
}
private static int lls(String S0, int k0, String S1, int k1, int[][] memo) {
    if (k0 == 0 || k1 == 0) return 0;
    if (memo[k0][k1] == -1) {
        if (S0[k0-1] == S1[k1-1])
            memo[k0][k1] = 1 + lls(S0, k0-1, S1, k1-1, memo);
        else
            memo[k0][k1] = Math.max(lls(S0, k0-1, S1, k1, memo),
                                    lls(S0, k0, S1, k1-1, memo));
    }
    return memo[k0][k1];
}
```

Q: How fast will the memoized version be? $\Theta(k_0 \cdot k_1)$

Side Trip into Java: Enumeration Types

- Problem: Need a type with a few, named, discrete values.
- In the purest form, the only necessary operations are == and !=; the only property of a value of the type is that it differs from all others.
- In older versions of Java, used named integer constants:

```
interface Pieces {  
    int BLACK_PIECE = 0,    // Fields in interfaces are static final.  
        BLACK_KING = 1,  
        WHITE_PIECE = 2,  
        WHITE_KING = 3,  
        EMPTY = 4;  
}
```

- C and C++ provide *enumeration types* as a shorthand, with syntax like this:

```
enum Piece { BLACK_PIECE, BLACK_KING, WHITE_PIECE, WHITE_KING, EMPTY };
```

- But since all these values are basically ints, accidents can happen.

Enum Types in Java

- More modern versions of Java allow syntax like that of C or C++, but with more guarantees:

```
public enum Piece
    { BLACK_PIECE, BLACK_KING, WHITE_PIECE, WHITE_KING, EMPTY; }
```

- Defines Piece as a new reference type, a special kind of class type.
- The names BLACK_PIECE, etc., are static, final *enumeration constants* (or *enumerals*) of type PIECE.
- They are automatically initialized, and are the only values of the enumeration type that exist (cannot say `new Piece()`).
- Can safely use `==`, and also switch statements:

```
boolean isKing(Piece p) {
    switch (p) {
        case BLACK_KING: case WHITE_KING: return true;
        default: return false;
    }
}
```

Making Enumerals Available Elsewhere

- Enumerals like `BLACK_PIECE` are static members of a class, not classes.
- Therefore, unlike `C` or `C++`, their declarations are not automatically visible outside the enumeration class definition.
- So, in other classes, must write `Piece.BLACK_PIECE`, which can get annoying.
- However, with version 1.5, Java has *static imports*: to import all static definitions of class `checkers.Piece` (including enumerals), you write

```
import static checkers.Piece.*;
```

among the import clauses.

- Alas, *cannot* use this for enum classes in the anonymous package.

Operations on Enum Types

- Order of declaration of enumeration constants significant: `.ordinal()` gives the position (numbering from 0) of an enumeration value. Thus, `Piece.BLACK_KING.ordinal()` is 1.
- The array `Piece.values()` gives all the possible values of the type. Thus, you can write:

```
for (Piece p : Piece.values())  
    System.out.printf("Piece value #%d is %s%n", p.ordinal(), p);
```

- The static function `Piece.valueOf` converts a String into a value of type `Piece`. So `Piece.valueOf("EMPTY") == EMPTY`.

Fancy Enum Types

- Enums are classes. You can define all the extra fields, methods, and constructors you want.
- Constructors are used only in creating enumeration constants. The constructor arguments follow the constant name:

```
enum Piece {
    BLACK_PIECE(BLACK, false, "b"), BLACK_KING(BLACK, true, "B"),
    WHITE_PIECE(WHITE, false, "w"), WHITE_KING(WHITE, true, "W"),
    EMPTY(null, false, " ");

    private final Side color;
    private final boolean isKing;
    private final String textName;

    Piece(Side color, boolean isKing, String textName) {
        this.color = color; this.isKing = isKing; this.textName = textName;
    }
    Side color() { return color; }
    boolean isKing() { return isKing; }
    String textName() { return textName; }
}
```