CS61B Lecture #20

Today:

- Trees

Readings for Today: *Data Structures*, Chapter 5

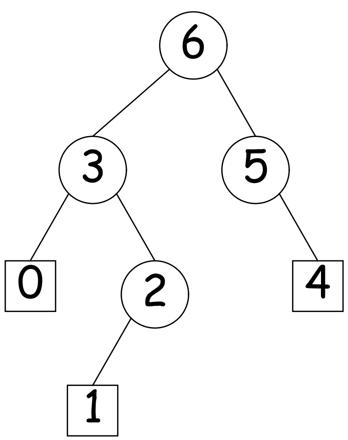
Readings for Next Topic: *Data Structures*, Chapter 6

A Recursive Structure

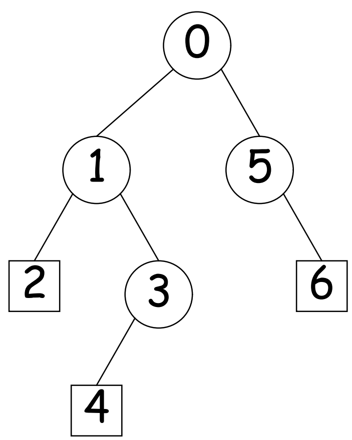
- Trees naturally represent recursively defined, hierarchical objects with more than one recursive subpart for each instance.
- Common examples: expressions, sentences.
  - Expressions have definitions such as "an expression consists of a literal or two expressions separated by an operator."
- Also describe structures in which we recursively divide a set into multiple subsets.

Fundamental Operation: Traversal

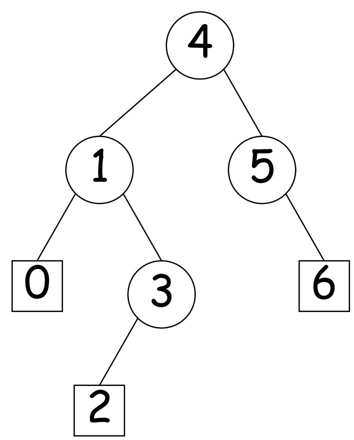
- *Traversing a tree* means enumerating (some subset of) its nodes.
- Typically done recursively, because that is natural description.
- As nodes are enumerated, we say they are *visited.*
- Three basic orders for enumeration (+ variations):
  - **Preorder:** visit node, traverse its children.
  - **Postorder:** traverse children, visit node.
  - **Inorder:** traverse first child, visit node, traverse second child (binary trees only).



Postorder



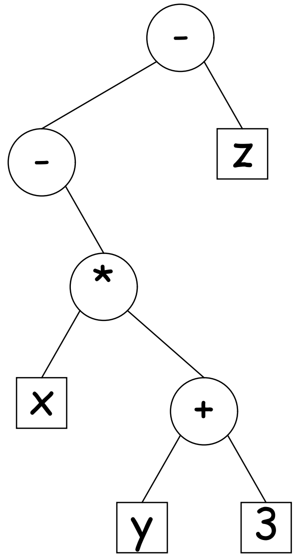
Preorder



inorder

Preorder Traversal and Prefix Expressions

Problem: Convert

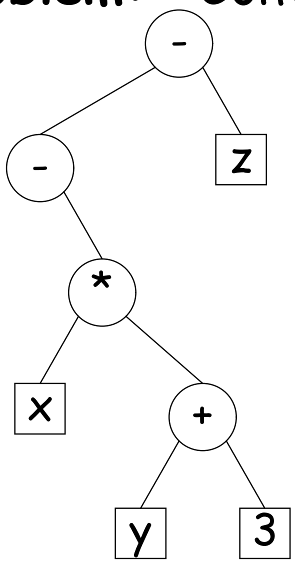


$\Rightarrow$ `(- (- (* x (+ y 3))) z)`

```
static String toLisp (Tree<String> T) {
  if (T == null)
    return "";
  else if (T.degree () == 0)
    return T.label ();
  else {
    String R;  R = "";
    for (int i = 0; i < T.numChildren (); i += 1)
      R += " " + toLisp (T.child (i));
    return String.format ("(%s%s)", T.label (), R);
  }
}
```

Inorder Traversal and Infix Expressions

Problem: Convert



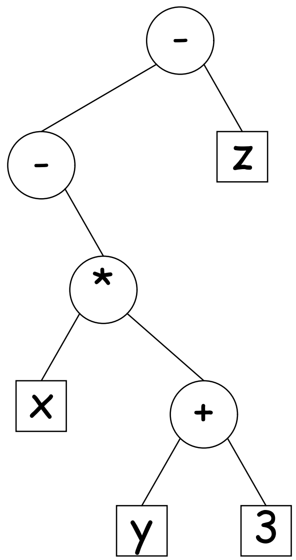
$\Rightarrow$ ((-(x*(y+3)))-z)

To think about: how to get rid of all those parentheses.

```
static String toInfix (Tree<String> T) {
  if (T == null)
    return "";
  if (T.degree () == 0)
    return T.label ();
  else {
    return String.format ("(%s%s%s)",
                          toInfix (T.left ()), T.label (), toInfix (T.right ())
  }
}
```

Postorder Traversal and Postfix Expressions

Problem: Convert



⇒ `x y 3 +:2 *:2 -:1 z -:2`

```
static String toPolish (Tree<String> T) {
  if (T == null)
    return "";
  else {
    String R;  R = "";
    for (int i = 0; i < T.numChildren (); i += 1)
      R += toPolish (T.child (i)) + " ";
    return String.format ("%s%s:%d", R, T.label (), T.degree ());
  }
}
```

A General Traversal: The Visitor Pattern

```
void preorderTraverse (Tree<Label> T, Action<Label> whatToDo)
{
  if (T != null) {
    whatToDo.action (T);
    for (int i = 0; i < T.numChildren (); i += 1)
      preorderTraverse (T.child (i), whatToDo);
  }
}
```

- What is `Action`?

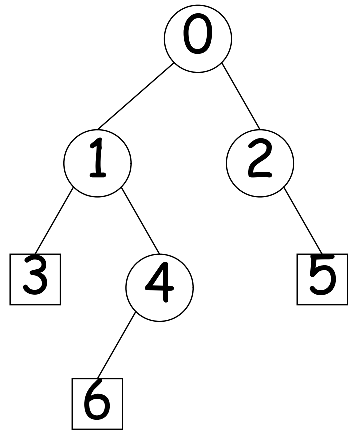
```
interface Action<Label> {
  void action (Tree<Label> T);
}

class Print implements Action<String> | preorderTraverse (myTree,
  void action (Tree<String> T) {      |                   new Print ());
    System.out.print (T.label ());    |
  }                                   |
}                                     |
```

Times

- The traversal algorithms have roughly the form of the `boom` example in §1.3.3 of *Data Structures*—an exponential algorithm.
- *However,* the role of M in that algorithm is played by the *height* of the tree, not the number of nodes.
- In fact, easy to see that tree traversal is *linear:* $\Theta(N)$, where N is the # of nodes: Form of the algorithm implies that there is one visit at the root, and then one visit for every *edge* in the tree. Since every node but the root has exactly one parent, and the root has none, must be $N - 1$ edges in any non-empty tree.
- In positional tree, is also one recursive call for each empty tree, but # of empty trees can be no greater than kN, where k is arity.
- For k-ary tree (max # children is k), $h + 1 \leq N \leq \frac{k^{h+1}-1}{k-1}$, where h is height.
- So $h \in \Omega(\log_k N) = \Omega(\lg N)$ and $h \in O(N)$.
- Many tree algorithms look at one child only. For them, time is proportional to the *height* of the tree, and this is $\Theta(\lg N)$, assuming that tree is *bushy*—each level has about as many nodes as possible.

Level-Order (Breadth-First) Traversal

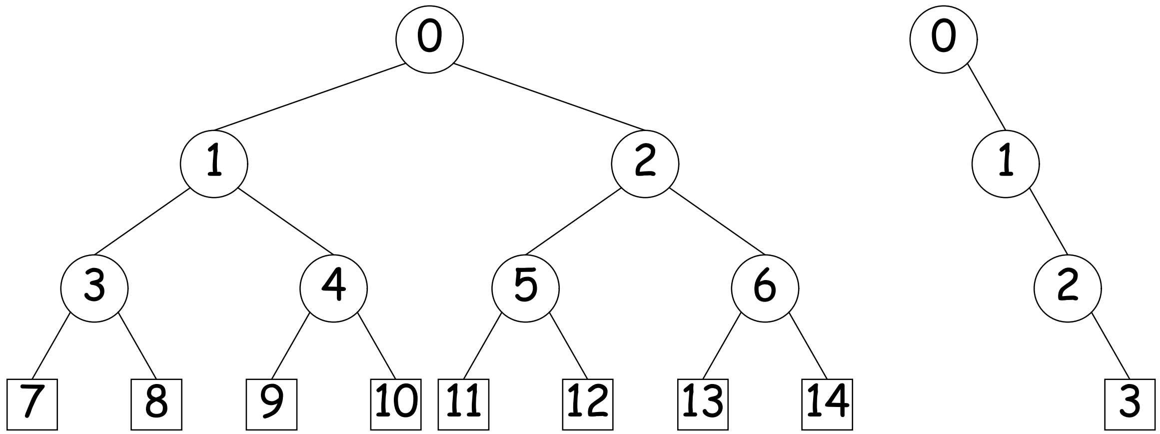
Problem: Traverse all nodes at depth 0, then depth 1, etc:



- One technique: *Iterative Deepening*. For each level, k, from 0 to h, call `doLevel(T,k)`

```
void doLevel (Tree T, int lev) {
  if (lev == 0)
    visit T
  else
    for each non-null child, C, of T {
      doLevel (C, lev-1);
    }
}
```

Iterative Deepening Time?



- Let h be height, N be # of nodes.
- Count # edges traversed (i.e, # of calls, not counting null nodes).
- First (full) tree: 1 for level 0, 3 for level 1, 7 for level 2, 15 for level 3.
- Or in general $(2^1 - 1) + (2^2 - 1) + \ldots + (2^{h+1} - 1) = 2^{h+2} - h \in \Theta(N)$, since $N = 2^{h+1} - 1$ for this tree.
- Second (*right leaning*) tree: 1 for level 0, 2 for level 2, 3 for level 3.
- Or in general $(h+1)(h+2)/2 = N(N+1)/2 \in \Theta(N^2)$, since $N = h+1$ for this kind of tree.

Iterative Traversals

- Tree recursion conceals data: a *stack* of nodes (all the `T` arguments) and a little extra information. Can make the data explicit, *e.g.*:

```
void preorderTraverse2 (Tree T<T>, Action whatToDo) {
  Stack s = new Stack ();
  s.push (T);
  while (! s.isEmpty ()) {
    Tree node = (Tree) s.pop ();
    if (node == null)
      continue;
    whatToDo.action (node);
    for (int i = node.numChildren ()-1; i >= 0; i -= 1)
      s.push (node.child (i));
  }
}
```

- To do a breadth-first traversal, use a queue instead of a stack, replace `push` with `add`, and `pop` with `removeFirst`.
- Makes breadth-first traversal worst-case linear time in all cases, but also linear *space* for "bushy" trees.

Iterators for Trees

- Frankly, iterators are not terribly convenient on trees.
- But can use ideas from iterative methods.

```
class PreorderTreeIterator<T> implements Iterator<T> {
  private Stack<Tree<T>> s = new Stack<Tree<T>> ();

  public PreorderTreeIterator (Tree<T> T) { s.push (T);  }

  public boolean hasNext () { return ! s.isEmpty (); }
  public T next () {
    Tree<T> result = s.pop ();
    for (int i = result.numChildren ()-1; i >= 0; i -= 1)
      s.push (result.child (i));
    return result.label ();
  }
  void remove () { throw new UnsupportedOperationException (); }
}
```

Example: (what do I have to add to class `Tree` first?)

```
for (String label : aTree) System.out.print (label + " ");
```