

1 Asymptotic analysis

In the following questions, give the runtime of the following functions in $\Theta(\cdot)$ or $O(\cdot)$ notation as requested. Your answer should be a function of N that is as simple as possible with no unnecessary leading constants or lower order terms. (MT2, Spring 2015)

(a) `public static void f1(int n) {` Runtime: $__\Theta(\quad)___$
`for (int i = 0; i < 2*n; i += 1) {`
`System.out.println("hello");`
`}`
`}`

(b) `public static void f3(int n) {` Runtime: $__\Theta(\quad)___$
`if (n == 0) { return; }`
`f3(n/3);`
`f1(n);`
`f3(n/3);`
`f1(n);`
`f3(n/3);`
`}`

(c) `public static void f4(int n) {` Runtime: $__\Theta(\quad)___$
`if (n == 0) { return; }`
`f4(n-1);`
`f1(17);`
`f4(n-1);`
`}`

(d) `public static void f5(int n, int m) {` Runtime: $__\Theta(\quad)___$
`if (m <= 0) {`
`return;`
`} else {`
`for (int i = 0; i < n; i += 1) {`
`f5(n, m-1);`
`}`
`}`
`}`

(e) `public static void f6(int n) {` Runtime: $__\Theta(\quad)___$
`if (n == 0) { return; }`
`f6(n / 2);`
`f6(n / 2);`
`f6(n / 2);`
`f6(n / 2);`
`g(n); // runs in $\Theta(N^2)$ time`
`}`

(f) `public static void f7(int n, int m) {` Runtime: $__O(\quad)__$
 `if (n < 10) { return; }`
 `for (int i = 0; i <= n % 10; i++) {`
 `f7(n / 10, m / 10);`
 `System.out.println(m);`
 `}`
`}`

2 More analysis

For each problem, give the best and worst-case runtimes in $\Theta(\cdot)$ notation as a function of N (or M). Your answer should be simple with no unnecessary leading constants or summations.

(a) `public static void removeIndex(int[] arr, int i) {`
 `// Assume i > 0`
 `int N = arr.length;`
 `for (int j = i; j < N; j += 1) {`
 `arr[j - 1] = arr[j];`
 `}`
`}`
 Best Case: $__O(\quad)__$ Worst Case: $__O(\quad)__$

(b) `public static int recurse(int N) {`
 `return helper(N, N / 2);`
`}`
`private static int helper(int N, int M) {`
 `if (N <= 1) {`
 `return N;`
 `}`
 `for (int i = 1; i < M; i *= 2) {`
 `System.out.println(i);`
 `}`
 `return helper(N - 1, M) + helper(N - 1, M);`
`}`
 Best Case: $__O(\quad)__$ Worst Case: $__O(\quad)__$

(c) `public static boolean find(int tgt, int[] arr) {`
 `int N = arr.length;`
 `return find(tgt, arr, 0, N);`
`}`
`private static boolean find(int tgt, int[] arr, int lo, int hi) {`
 `if (lo == hi || lo + 1 == hi) {`
 `return arr[lo] == tgt;`
 `}`
 `int mid = (lo + hi) / 2;`
 `for (int i = 0; i < mid; i += 1) {`
 `System.out.println(arr[i]);`
 `}`
 `return arr[mid] == tgt || find(tgt, arr, lo, mid) || find(tgt, arr,`
 `mid, hi);`
`}`
 Best Case: $__O(\quad)__$ Worst Case: $__O(\quad)__$

3 Bit Operations

In the following questions, use bit manipulation operations to achieve the intended functionality and fill out the function details -

- (a) Implement a function `isPalindrome` which checks if the binary representation of a given number is palindrome. The function returns true if and only if the binary representation of `num` is a palindrome.

For example, the function should return true for `isPalindrome(9)` since binary representation of 9 is 1001 which is a palindrome.

```
/**
 * Returns true if binary representation of num is a palindrome
 */
public static boolean isPalindrome(int num)
{
    // stores reverse of binary representation of num
    int reverse = 0;

    _____
    _____
    _____
    _____
    _____
    _____
    _____

    return num == reverse;
}
```

- (b) Implement a function `swap` which for a given integer, swaps two bits at given positions. The function returns the resulting integer after bit swap operation.

For example, when the function is called with inputs `swap(31, 3, 7)`, it should reverse the 3rd and 7th bits from the right and return 91 since 31 (00011111) would become 91 (01011011).

```
/**
 * Function to swap bits at position a and b (from right) in integer num
 */
public static int swap(int num, int a, int b)
{
    _____
    _____
    _____
    _____
    _____
    _____
    _____

    return num;
}
```