

## 1 Asymptotics is Fun!

- (a) Using the function `g` defined below, what is the runtime of the following function calls? Write each answer in terms of `N`.

```
1 void g(int N, int x) {
2     if (N == 0) {
3         return;
4     }
5     for (int i = 1; i <= x; i++) {
6         g(N - 1, i);
7     }
8 }
```

`g(N, 1):  $\Theta(\quad)$`

`g(N, 2):  $\Theta(\quad)$`

- (b) Suppose we change line 6 to `g(N - 1, x)` and change the stopping condition in the for loop to `i <= f(x)` where `f` returns a random number between 1 and `x`, inclusive. For the following function calls, find the tightest  $\Omega$  and big  $O$  bounds.

```
1 void g(int N, int x) {
2     if (N == 0) {
3         return;
4     }
5     for (int i = 1; i <= f(x); i++) {
6         g(N - 1, x);
7     }
8 }
```

`g(N, 2):  $\Omega(\quad)$ ,  $O(\quad)$`

`g(N, N):  $\Omega(\quad)$ ,  $O(\quad)$`

## 2 Slightly Harder (Spring 2017, MT2)

Give the runtime of the following functions in  $\Theta$  or  $O$  notation as requested. Your answer should be as simple as possible with no unnecessary leading constants or lower order terms. For f5, your bound should be as tight as possible (so don't just put  $O(N^{NM!})$  or similar for the second answer).

```

1 public static void f4(int N) {
2     if (N == 0) {return;}
3     f4(N / 2);
4     f4(N / 2);
5     f4(N / 2);
6     f4(N / 2);
7     g(N); // runs in  $\Theta(N^2)$  time
8 }
```

Runtime:  $\Theta(\quad)$

```

1 public static void f5(int N, int M) {
2     if (N < 10) {return;}
3     for (int i = 0; i <= N % 10; i++) {
4         f5(N / 10, M / 10);
5         System.out.println(M);
6     }
7 }
```

Runtime:  $O(\quad)$

### 3 Flip Flop

Suppose we have the `flip` function as defined below. Assume the method `unknown` returns a random integer between 1 and  $N$ , exclusive, and runs in constant time. For each definition of the `flop` method below, give the best and worst case runtime of `flip` in  $\Theta(\cdot)$  notation as a function of  $N$ .

```

1 public static void flip(int N) {
2     if (N <= 100) {
3         return;
4     }
5     int stop = unknown(N);
6     for (int i = 1; i < N; i++) {
7         if (i == stop) {
8             flop(i, N);
9             return;
10        }
11    }
12 }
```

(a) `public static void flop(int i, int N) {`  
`flip(N - i);`  
`}`

Best Case:  $\Theta(\quad)$ , Worst Case:  $\Theta(\quad)$

(b) `public static void flop(int i, int N) {`  
`int minimum = Math.min(i, N - i);`  
`flip(minimum);`  
`flip(minimum);`  
`}`

Best Case:  $\Theta(\quad)$ , Worst Case:  $\Theta(\quad)$

(c) `public static void flop(int i, int N) {`  
`flip(i);`  
`flip(N - i);`  
`}`

Best Case:  $\Theta(\quad)$ , Worst Case:  $\Theta(\quad)$