

CS61B Lecture #24

Today:

- Priority queues (*Data Structures* §6.4, §6.5)
- Range queries (§6.2)
- Java utilities: SortedSet, Map, etc.

Priority Queues, Heaps

- Priority queue: defined by operations “add,” “find largest,” “remove largest.”
- Examples: scheduling long streams of actions to occur at various future times.
- Also useful for sorting (keep removing largest).
- Common implementation is the *heap*, a kind of tree.
- (Confusingly, this same term is used to describe the pool of storage that the *new* operator uses. Sorry about that.)

Heaps

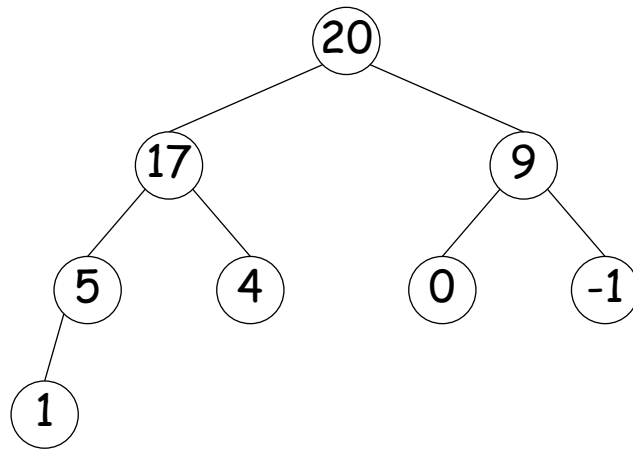
- A *max-heap* is a binary tree that enforces the *Heap Property*: Labels of *both* children of each node are less than node's label.
 - So the node at top has largest label.
 - Looser than binary search property, which allows us to keep tree "bushy".
 - That is, it's always valid to put the smallest nodes anywhere at the bottom of the tree.
 - Thus, heaps can be made *nearly complete*: all but possibly the last row have as many keys as possible.
 - As a result, insertion of new value and deletion of largest value always take time proportional to $\lg N$ in worst case.
- A *min-heap* is basically the same, but with the minimum value at the root and children having larger values than their parents.

Example: Inserting into a simple heap

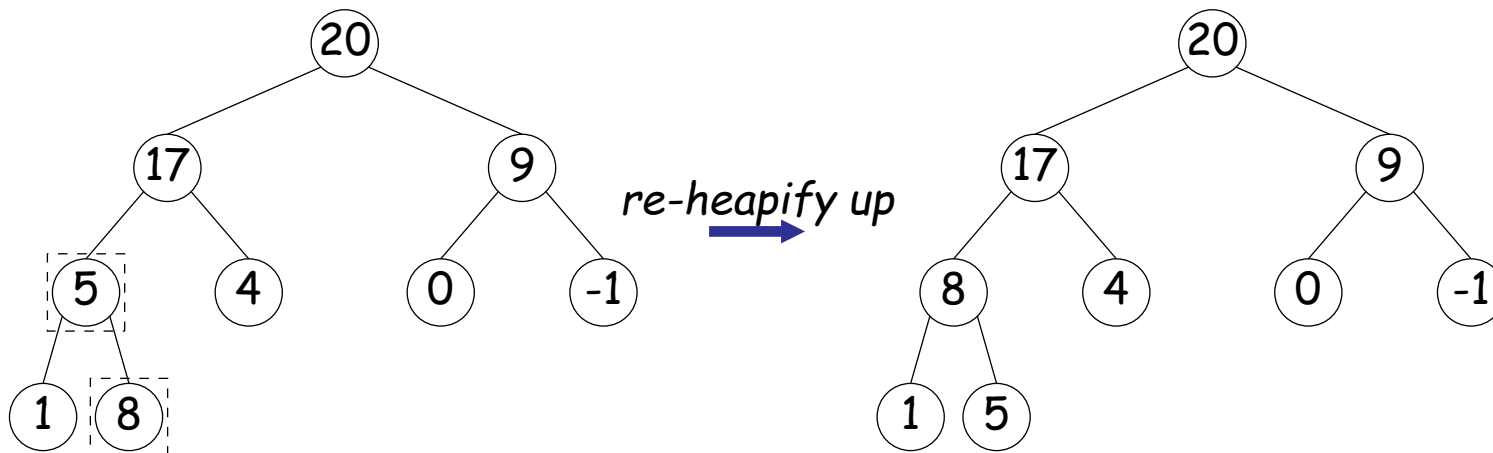
Data:

1 17 4 5 9 0 -1 20

Initial Heap:

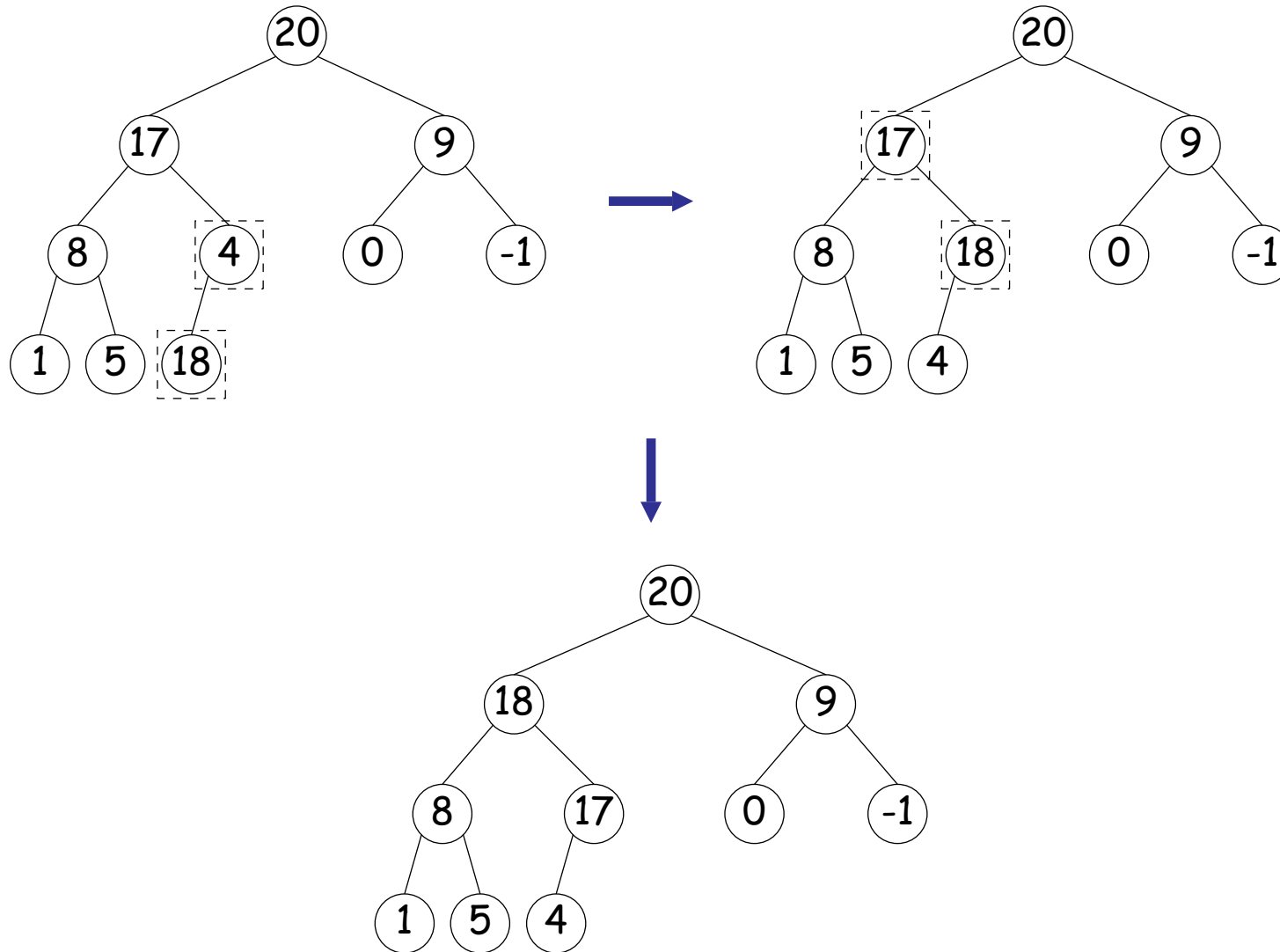


Add 8: Dashed boxes show where the heap property is violated



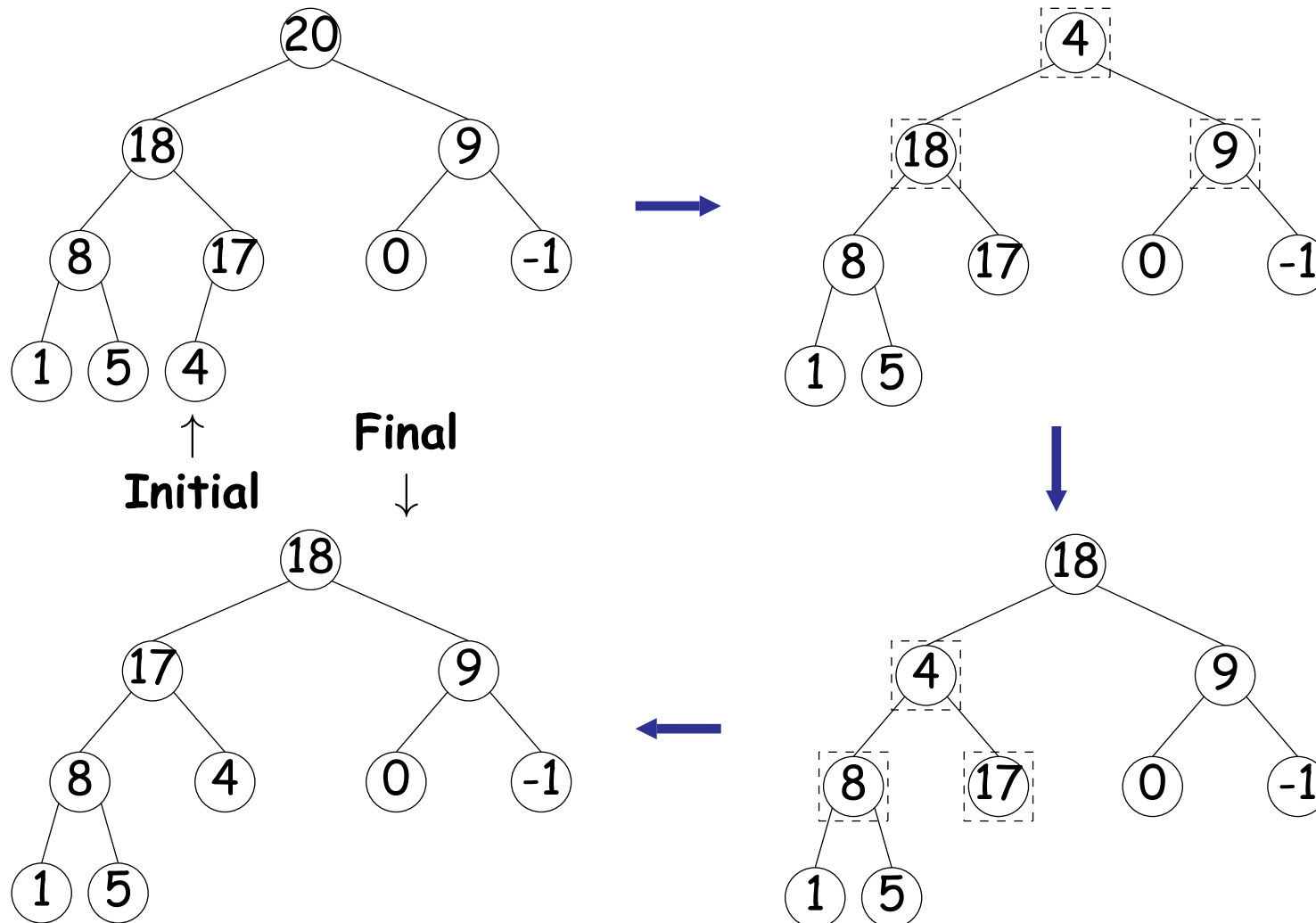
Heap insertion continued

Now insert 18:



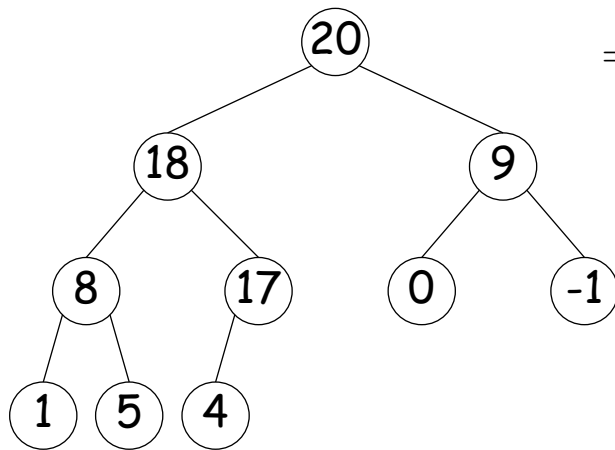
Removing Largest from Heap

To remove largest: Move the bottommost, rightmost node to the top, then re-heapify down as needed (swap offending node with larger child) to re-establish the heap property.

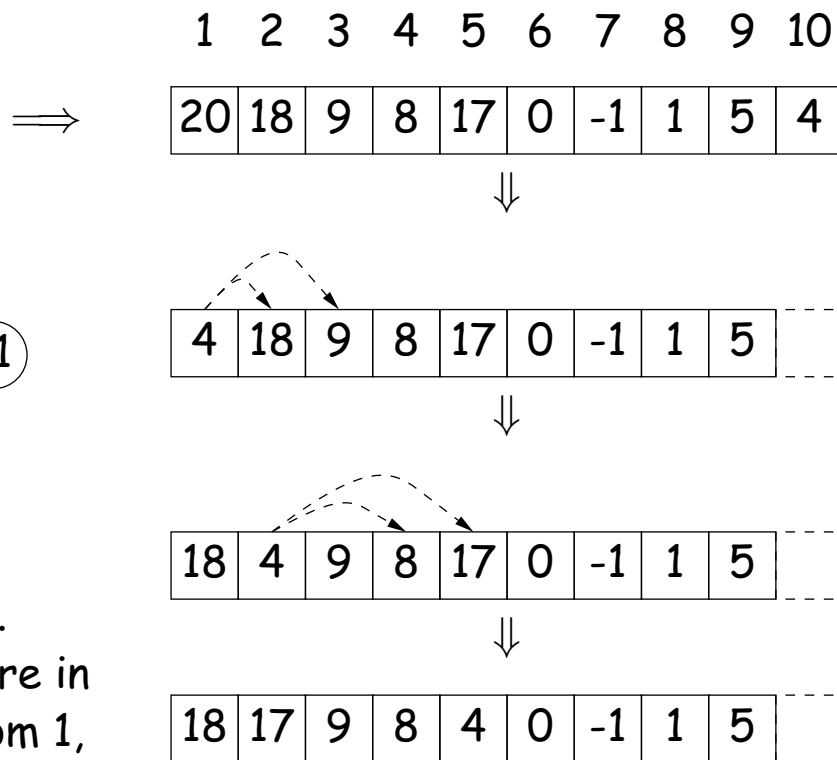


Heaps in Arrays

- Since heaps are nearly complete (missing items only at bottom level), can use arrays for compact representation.
- Example of removal from last slide (dashed arrows show children):



Nodes stored in level order.
 Children of node at index $#K$ are in $2K$ and $2K + 1$ if numbering from 1, or $2K + 1$ and $2K + 2$ if from 0.



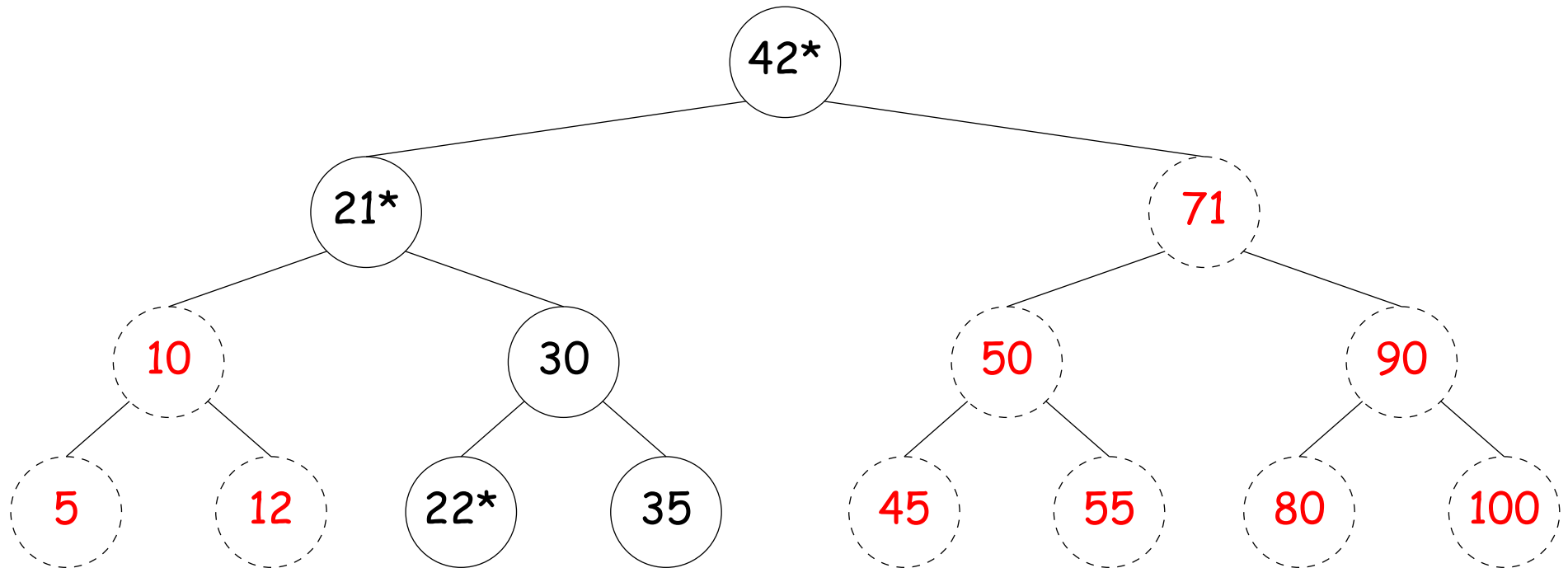
Ranges

- So far, we have looked for specific items,
- But for BSTs, we need an ordering anyway, and we can also support looking for *ranges of values*.
- Example: perform some action on all values in a BST that are within some range (in natural order):

```
/** Apply WHATTODO to all labels in T that are >= L and < U,
 * in ascending natural order. */
static void visitRange(BST<String> T, String L, String U,
                      Consumer<BST<String>> whatToDo) {
    if (T != null) {
        int compLeft = L.compareTo(T.label ()),
            compRight = U.compareTo(T.label ());
        if (compLeft < 0) /* L < label */
            visitRange (T.left(), L, U, whatToDo);
        if (compLeft <= 0 && compRight > 0) /* L <= label < U */
            whatToDo.accept(T);
        if (compRight > 0) /* label < U */
            visitRange (T.right (), L, U, whatToDo);
    }
}
```


Time for Range Queries

- Time for range query $\in O(h + M)$, where h is the height of the tree, and M is the number of data items that turn out to be in the range.
- Consider searching the tree below for all values $25 \leq x < 40$.
- **Dashed** nodes are never looked at. Starred nodes are looked at but not output. The h comes from the starred nodes; the M comes from unstarred non-dashed nodes.



Ordered Sets and Range Queries in Java

- Class `SortedSet` supports range queries with *views* of a set:
 - `S.headSet(U)`: subset of `S` that is $< U$.
 - `S.tailSet(L)`: subset that is $\geq L$.
 - `S.subSet(L,U)`: subset that is $\geq L, < U$.
- Changes to views modify `S`.
- Attempts to, e.g., add to a `headSet` beyond `U` are disallowed.
- Can iterate through a view to process a range:

```
SortedSet<String> fauna = new TreeSet<String>
    (Arrays.asList ("axolotl", "elk", "dog", "hartebeest", "duck"));
for (String item : fauna.subSet ("bison", "gnu"))
    System.out.printf ("%s, ", item);
```

would print "dog, duck, elk,"

TreeSet

- Java library type `TreeSet<T>` requires either that `T` be `Comparable`, or that you provide a `Comparator`, as in:

```
SortedSet<String> rev_fauna = new TreeSet<String>(Collections.reverseOrder());
```

- `Comparator` is a type of function object:

```
interface Comparator<T> {  
    /** Return <0 if LEFT<RIGHT, >0 if LEFT>RIGHT, else 0. */  
    int compare(T left, T right);  
}
```

(We'll deal with what `Comparator<T extends Comparable<T>>` is all about later.)

- For example, the `reverseOrder` comparator is defined like this:

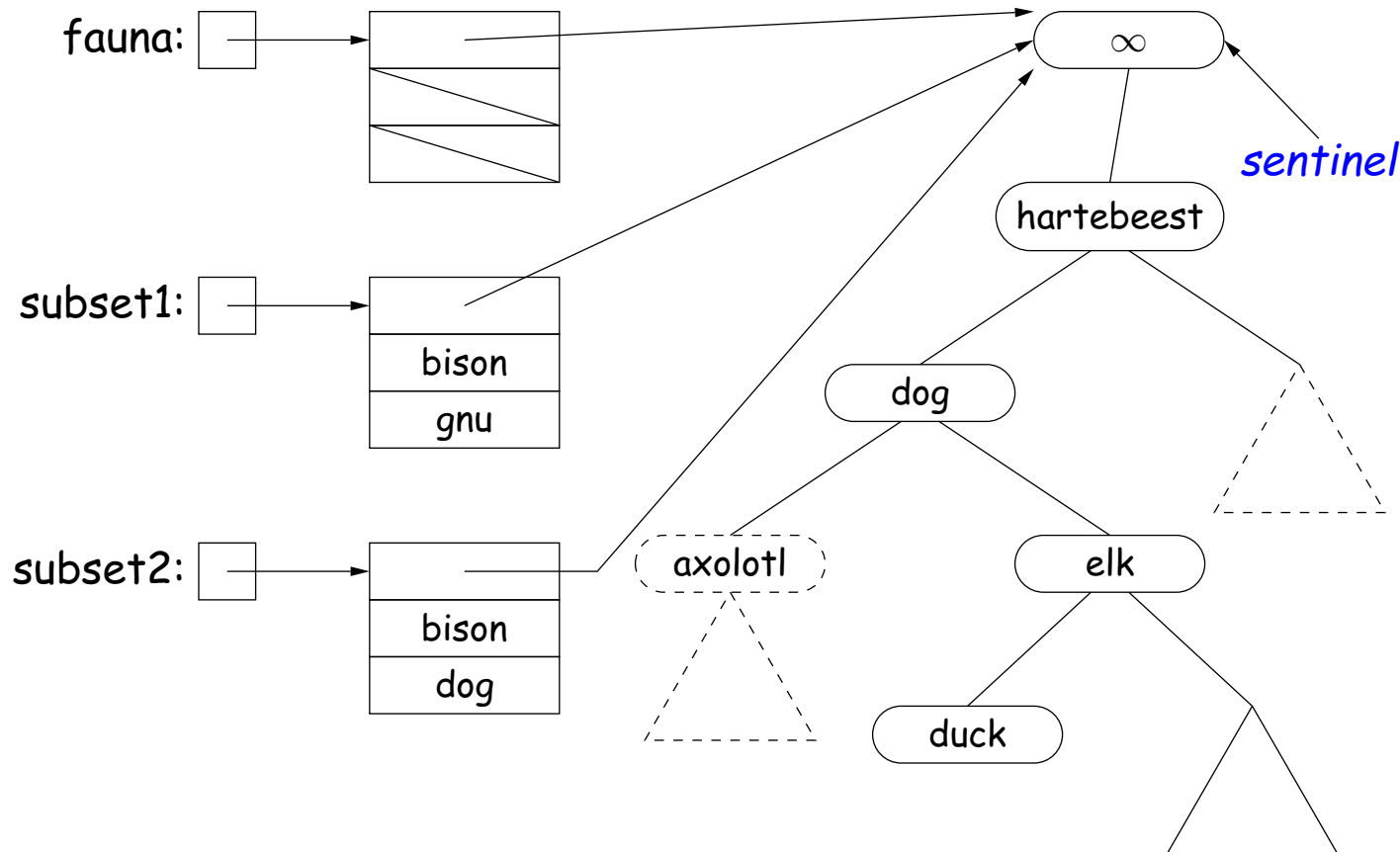
```
/** A Comparator that gives the reverse of natural order. */  
static <T extends Comparable<T>> Comparator<T> reverseOrder() {  
    // Java figures out this lambda expression is a Comparator<T>.  
    return (x, y) -> y.compareTo(x);  
}
```

Example of Representation: BSTSet

- Same representation for both sets and subsets.
- Pointer to BST, plus bounds (if any).
- `.size()` is expensive!

```
SortedSet<String>
```

```
fauna = new BSTSet<String>(stuff);  
subset1 = fauna.subSet("bison", "gnu");  
subset2 = subset1.subSet("axolotl", "dog");
```



Comparing Search Structures

Here, N is #items, k is #answers to query.

Function	Unordered List	Sorted Array	Bushy Search Tree	"Good" Hash Table	Heap
<i>find</i>	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(N)$
<i>add (amortized)</i>	$\Theta(1)$	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(\lg N)$
<i>range query</i>	$\Theta(N)$	$\Theta(k + \lg N)$	$\Theta(k + \lg N)$	$\Theta(N)$	$\Theta(N)$
<i>find largest</i>	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(1)$
<i>remove largest</i>	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(\lg N)$