Today:

- Priority queues (Data Structures §6.4, §6.5)
- Range queries (§6.2)
- Java utilities: SortedSet, Map, etc.
Priority Queues, Heaps

- Priority queue: defined by operations “add,” “find largest,” “remove largest.”
- Examples: scheduling long streams of actions to occur at various future times.
- Also useful for sorting (keep removing largest).
- Common implementation is the heap, a kind of tree.
- (Confusingly, this same term is used to described the pool of storage that the new operator uses. Sorry about that.)
Heaps

- A **max-heap** is a binary tree that enforces the *Heap Property*: Labels of both children of each node are less than node’s label.

- So the node at top has largest label.

- Looser than binary search property, which allows us to keep tree “bushy”.

- That is, it’s always valid to put the smallest nodes anywhere at the bottom of the tree.

- Thus, heaps can be made *nearly complete*: all but possibly the last row have as many keys as possible.

- As a result, insertion of new value and deletion of largest value always take time proportional to $\log N$ in worst case.

- A **min-heap** is basically the same, but with the minimum value at the root and children having larger values than their parents.
Example: Inserting into a simple heap

Data:
1 17 4 5 9 0 -1 20

Initial Heap:

```
  20
 /   \
17   9
 / \
5  4 0 -1
 / \
1  8
```

Add 8: Dashed boxes show where the heap property is violated

Add 8:

```
  20
 /   \
17   9
 / \
5  4 0 -1
 / \
1  8
```

```
  20
 /   \
17   9
 / \
8   4 0 -1
 / \
1  5
```

re-heapify up
Heap insertion continued

Now insert 18:
Removing Largest from Heap

To remove largest: Move the bottommost, rightmost node to the top, then re-heapify down as needed (swap offending node with larger child) to re-establish the heap property.
Heaps in Arrays

- Since heaps are nearly complete (missing items only at bottom level), can use arrays for compact representation.

- Example of removal from last slide (dashed arrows show children):

Nodes stored in level order. Children of node at index #K are in 2K and 2K + 1 if numbering from 1, or 2K + 1 and 2K + 2 if from 0.
Ranges

- So far, we have looked for specific items,
- But for BSTs, we need an ordering anyway, and we can also support looking for *ranges of values*.
- Example: perform some action on all values in a BST that are within some range (in natural order):

```java
/** Apply WHATTODO to all labels in T that are >= L and < U, in ascending natural order. */
static void visitRange(BST<String> T, String L, String U, Consumer<BST<String>> whatToDo) {
    if (T != null) {
        int compLeft = L.compareTo(T.label ()),
            compRight = U.compareTo(T.label ());
        if (compLeft < 0) /* L < label */
            visitRange (T.left(), L, U, whatToDo);
        if (compLeft <= 0 && compRight > 0) /* L <= label < U */
            whatToDo.accept(T);
        if (compRight > 0) /* label < U */
            visitRange (T.right (), L, U, whatToDo);
    }
}
```
Time for Range Queries

- Time for range query $\in O(h + M)$, where $h$ is the height of the tree, and $M$ is the number of data items that turn out to be in the range.

- Consider searching the tree below for all values $25 \leq x < 40$.

- Dashed nodes are never looked at. Starred nodes are looked at but not output. The $h$ comes from the starred nodes; the $M$ comes from unstarrred non-dashed nodes.
Ordered Sets and Range Queries in Java

- **Class** SortedSet supports range queries with *views* of a set:
  - S.headSet(U): subset of S that is \(< U\).
  - S.tailSet(L): subset that is \(\geq L\).
  - S.subSet(L,U): subset that is \(\geq L, < U\).

- Changes to views modify S.

- Attempts to, e.g., add to a headSet beyond \(U\) are disallowed.

- Can iterate through a view to process a range:

```java
SortedSet<String> fauna = new TreeSet<String>(Arrays.asList("axolotl", "elk", "dog", "hartebeest", "duck"));
for (String item : fauna.subSet("bison", "gnu"))
    System.out.printf("%s, ", item);
```

would print “dog, duck, elk,”
TreeSet

- Java library type TreeSet<T> requires either that T be Comparable, or that you provide a Comparator, as in:

  ```java
  SortedSet<String> rev_fauna = new TreeSet<String>(Collections.reverseOrder());
  ```

- Comparator is a type of function object:

  ```java
  interface Comparator<T> {
    /** Return <0 if LEFT<RIGHT, >0 if LEFT>RIGHT, else 0. */
    int compare(T left, T right);
  }
  ```

  (We’ll deal with what Comparator<T extends Comparable<T>> is all about later.)

- For example, the reverseOrder comparator is defined like this:

  ```java
  /** A Comparator that gives the reverse of natural order. */
  static <T extends Comparable<T>> Comparator<T> reverseOrder() {
    // Java figures out this lambda expression is a Comparator<T>.
    return (x, y) -> y.compareTo(x);
  }
  ```
Example of Representation: BSTSet

- Same representation for both sets and subsets.
- Pointer to BST, plus bounds (if any).
- `.size()` is expensive!

```java
SortedSet<String>
fauna = new BSTSet<String>(stuff);
subset1 = fauna.subSet("bison","gnu");
subset2 = subset1.subSet("axolotl","dog");
```
Comparing Search Structures

Here, $N$ is #items, $k$ is #answers to query.

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<th>Sorted Array</th>
<th>Bushy Search Tree</th>
<th>“Good” Hash Table</th>
<th>Heap</th>
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<td>find</td>
<td>$\Theta(N)$</td>
<td>$\Theta(lg;N)$</td>
<td>$\Theta(lg;N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(N)$</td>
</tr>
<tr>
<td>add (amortized)</td>
<td>$\Theta(1)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(lg;N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(lg;N)$</td>
</tr>
<tr>
<td>range query</td>
<td>$\Theta(N)$</td>
<td>$\Theta(k + lg;N)$</td>
<td>$\Theta(k + lg;N)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(N)$</td>
</tr>
<tr>
<td>find largest</td>
<td>$\Theta(N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(lg;N)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(1)$</td>
</tr>
<tr>
<td>remove largest</td>
<td>$\Theta(N)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(lg;N)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(lg;N)$</td>
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