CS61B Lecture #31

Today:

- More balanced search structures ($DS(IJ)$, Chapter 9)
Really Efficient Use of Keys: the Trie

- We haven’t said much about the cost of comparisons, generally treating the cost as constant.
- For strings, the worst case is the length of string.
- Therefore we should throw an extra factor of the key length, $L$, into costs:
  - $\Theta(M)$ comparisons really means $\Theta(ML)$ operations.
  - So to look for key $X$, we keep looking at same chars of $X$ $M$ times.
- Can we do better? Can we get search cost to be $O(L)$?

Idea: Make a multi-way decision tree, with one decision per character of key.
The Trie: Example

- Set of keys
  \{a, abase, abash, abate, abbas, axolotl, axe, fabric, facet\}
- Ticked lines show paths followed for “abash” and “fabric”
- Each internal node corresponds to a possible prefix.
- Characters in path to node = that prefix.
Adding Item to a Trie

- Result of adding bat and faceplate.
- New edges ticked.

![Trie Diagram]

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A Side-Trip: Scrunching

• For speed, obvious implementation for internal nodes is array indexed by character.

• Gives $O(L)$ performance, $L$ length of search key.

• [Looks as if independent of $N$, number of keys. Is there a dependence?]

• Problem: arrays are *sparsely populated* by non-null values—waste of space.

Idea: Put the arrays on top of each other!

• Use null (0, empty) entries of one array to hold non-null elements of another.

• Use extra markers to tell which entries belong to which array.
Scrunching Example

Small example: (unrelated to Tries on preceding slides)

- Three arrays, each indexed 0..9

<table>
<thead>
<tr>
<th>A1:</th>
<th>A2:</th>
<th>A3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2*</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5*</td>
<td>5*</td>
<td>5*</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6*</td>
</tr>
<tr>
<td>7*</td>
<td>7*</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8*</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>9*</td>
</tr>
</tbody>
</table>

- Now overlay them, but keep track of the original index of each item:

<table>
<thead>
<tr>
<th>A3:</th>
<th>A2:</th>
<th>A1:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0*</td>
</tr>
<tr>
<td>1*</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2*</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5*</td>
<td>5*</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6*</td>
</tr>
<tr>
<td>7</td>
<td>7*</td>
<td>7*</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8*</td>
</tr>
<tr>
<td>9*</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Check:

<table>
<thead>
<tr>
<th>A123:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>-1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>-1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>5*</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7*</td>
</tr>
<tr>
<td>9*</td>
</tr>
</tbody>
</table>

Starred items are null in uncompressed array

Index in original array or -1 if null in all arrays

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Scrunching Example (contd.)

A1:

\[\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
\text{bass} & & & & & & & & & \\
\text{trout} & & & & & & & & & \\
\text{pike} & & & & & & & & & \\
\end{array}\]

A2:

\[\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
\text{ghee} & \text{milk} & \text{oil} & \text{milk} & \text{oil} & \text{milk} & \text{oil} & \text{milk} & \text{oil} & \text{milk} \\
\end{array}\]

A3:

\[\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
\text{salt} & \text{cumin} & \text{mace} & \text{mace} & \text{mace} & \text{mace} & \text{mace} & \text{mace} & \text{mace} & \text{mace} \\
\end{array}\]

Check:

\[\begin{array}{cccccccccc}
0 & -1 & 1 & -1 & 2 & 5 & 5 & 7 & 6 & 7 & 9 \\
\end{array}\]

A123:

\[\begin{array}{cccccccccc}
\text{bass} & \text{ghee} & \text{pike} & \text{salt} & \text{cumin} & \text{mace} & \text{mace} & \text{mace} & \text{mace} & \text{mace} & \text{mace} \\
\end{array}\]

\[/* \ A1[i] == */ (Check[i] == i) \ ? \ A123[i] : null; /* \ A2[i] == */ (Check[i + 2] == i) \ ? \ A123[i + 2] : null; /* \ A3[i] == */ (Check[i + 1] == i) \ ? \ A123[i + 1] : null; \]
Practicum

• The scrunching idea is cute, but
  - Not so good if we want to expand our trie.
  - A bit complicated.
  - Actually more useful for representing large, sparse, fixed tables with many rows and columns.

• Furthermore, number of children in trie tends to drop drastically when one gets a few levels down from the root.

• So in practice, might as well use linked lists to represent set of node’s children...

• ...but use arrays for the first few levels, which are likely to have more children.
Probabilistic Balancing: Skip Lists

- A skip list can be thought of as a kind of n-ary search tree in which we choose to put the keys at “random” heights.

- More often thought of as an ordered list in which one can skip large segments.

- Typical example:

  ![Skip List Diagram]

  - To search, start at top layer on left, search until next step would overshoot, then go down one layer and repeat.

  - In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.

  - Heights of the nodes were chosen randomly so that there are about \( \frac{1}{2} \) as many nodes that are \( \geq k \) high as there are that are \( k \) high.

  - Makes searches fast with high probability.
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- Typical example:

\[ \begin{array}{cccccccccccc}
-\infty & 10 & 20 & 25 & 30 & 40 & 50 & 55 & 60 & 90 & 95 & 100 & 115 & 120 & 125 & 130 & 140 & 150 & \infty
\end{array} \]

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```

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- In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.
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- Typical example:

  ![Diagram of a skip list with values ranging from negative infinity to 150]

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- Typical example:

  ![Skip List Diagram](image)

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  - Makes searches fast with high probability.
Example: Adding and deleting

- Starting from initial list:

- In any order, we add 126 and 127 (choosing random heights for them), and remove 20 and 40:

- Shaded nodes here have been modified.
Summary

• Balance in search trees allows us to realize $\Theta(lg N)$ performance.

• B-trees, red-black trees:
  - Give $\Theta(lg N)$ performance for searches, insertions, deletions.
  - B-trees good for external storage. Large nodes minimize # of I/O operations

• Tries:
  - Give $\Theta(B)$ performance for searches, insertions, and deletions, where $B$ is length of key being processed.
  - But hard to manage space efficiently.

• Interesting idea: scrunched arrays share space.

• Skip lists:
  - Give probable $\Theta(lg N)$ performance for searches, insertions, deletions
  - Easy to implement.
  - Presented for interesting ideas: probabilistic balance, randomized data structures.
Summary of Collection Abstractions

**Multiset**
- contains, iterator

**List**
- get(n)

**Set**
- Ordered Set
  - first
- Unordered Set

**Priority Queue**

**Sorted Set**
- subset

**Map**
- contains, iterator
- get

**Unordered Map**

**Ordered Map**

**Key Colors**
- **Blue**: Java has corresponding interface
- **Green**: Java has no corresponding interface
Data Structures that Implement Abstractions

Multiset

- **List**: arrays, linked lists, circular buffers
- **Set**
  - **OrderedSet**
    - *Priority Queue*: heaps
    - *Sorted Set*: binary search trees, red-black trees, B-trees, sorted arrays or linked lists
  - **Unordered Set**: hash table

Map

- **Unordered Map**: hash table
- **Ordered Map**: red-black trees, B-trees, sorted arrays or linked lists
**Corresponding Classes in Java**

**Multiset** (Collection)
- **List**: ArrayList, LinkedList, Stack, ArrayBlockingQueue, ArrayDeque
- **Set**
  - **OrderedSet**
    - *Priority Queue*: PriorityQueue
    - *Sorted Set* (SortedSet): TreeSet
  - **Unordered Set**: HashSet

**Map**
- **Unordered Map**: HashMap
- **Ordered Map** (SortedMap): TreeMap