CS61B Lecture #36

Today:

- Dynamic Programming
- A Brief Side Trip: Enumeration types.

Dynamic Programming

- A puzzle (D. Garcia):
  - Start with a list with an even number of non-negative integers.
  - Each player in turn takes either the leftmost number or the rightmost.
  - Idea is to get the largest possible sum.
- Example: starting with (6, 12, 0, 8), you (as first player) should take the 8. Whatever the second player takes, you also get the 12, for a total of 20.
- Assuming your opponent plays perfectly (i.e., to get as much as possible), how can you maximize your sum?
- Can solve this with exhaustive game-tree search, but...

Obvious Program

- Recursion makes it easy, again:

```
int bestSum(int[] V) {
    int total, i, N = V.length;
    for (i = 0, total = 0; i < N; i += 1) total += V[i];
    return N == 0 ? 0 : bestSum(V, 0, N-1, total);
}

/** The largest sum obtainable by the first player in the game on
 *  the list V[LEFT..RIGHT], given TOTAL = sum of V[LEFT..RIGHT]. */
int bestSum(int[] V, int left, int right, int total) {
    if (left == right) {
        return V[left];
    } else {
        int L = total - bestSum(V, left+1, right, total-V[left]);
        int R = total - bestSum(V, left, right-1, total-V[right]);
        return Math.max(L, R);
    }
}
```

- Time cost is $C(0) = 1,\ C(N) = 2C(N-1)$; so $C(N) \in \Theta(2^N)$

Still Another Idea from CS61A

The problem is that we are recomputing intermediate results many times. Solution: *memoize* the intermediate results. Here, we pass in an $N \times N$ array ($N =$ `V.length`) of memoized results, all initially -1.

```
int bestSum(int[] V) {
    int total, i, N = V.length;
    int[] memo = new int[N][N];
    for (i = 0, total = 0; i < N; i += 1) total += V[i];
    for (int[] row : memo) Arrays.fill(row, -1);
    return N == 0 ? 0 : bestSum(V, 0, N-1, total, memo);
}

int bestSum(int[] V, int left, int right, int total, int[][] memo) {
    if (left == right) memo[left][right] = V[left];
    else if (memo[left][right] == -1) {
        int L = total - bestSum(V, left+1, right, total-V[left], memo);
        int R = total - bestSum(V, left, right-1, total-V[right], memo);
        memo[left][right] = Math.max(L, R);
    }
    return memo[left][right];
}
```

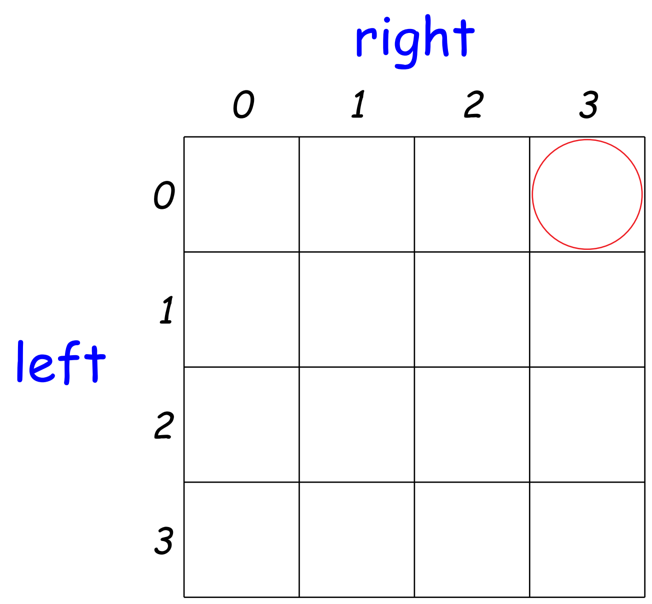
Time Cost with Memoization

```
int bestSum(int[] V, int left, int right, int total, int[][] memo) {
    if (left == right) memo[left][right] = V[left];
    else if (memo[left][right] == -1) {
        int L = total - bestSum(V, left+1, right, total-V[left], memo);
        int R = total - bestSum(V, left, right-1, total-V[right], memo);
        memo[left][right] = Math.max(L, R);
    }
    return memo[left][right];
}
```

- Now we will perform the full computation of each entry in `memo` once.
- The number of recursive calls to `bestSum` must be $O(N^2)$, for $N =$ the length of V, an enormous improvement from $\Theta(2^N)$!

Iterative Version

- In the usual presentation of this idea—known as *dynamic programming*—we figure out ahead of time the order in which the memoized version will fill in `memo`, and write an explicit loop, saving the time needed to ceck whether the result has already been computed.
- Suppose that we use as input `{ 2 1 8 3 }`. Our goal is to find the circled—(0,3)—entry in the memo table, which gets filled in like this (grayed boxes indicates table entries that are used to compute a new entry):



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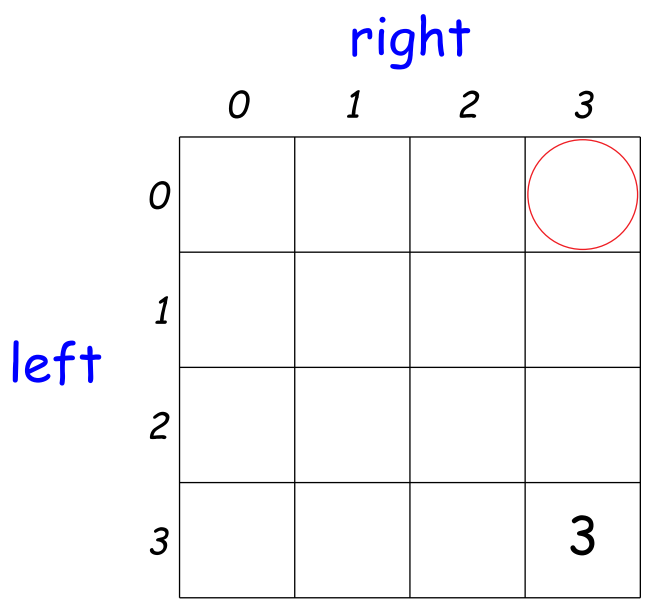

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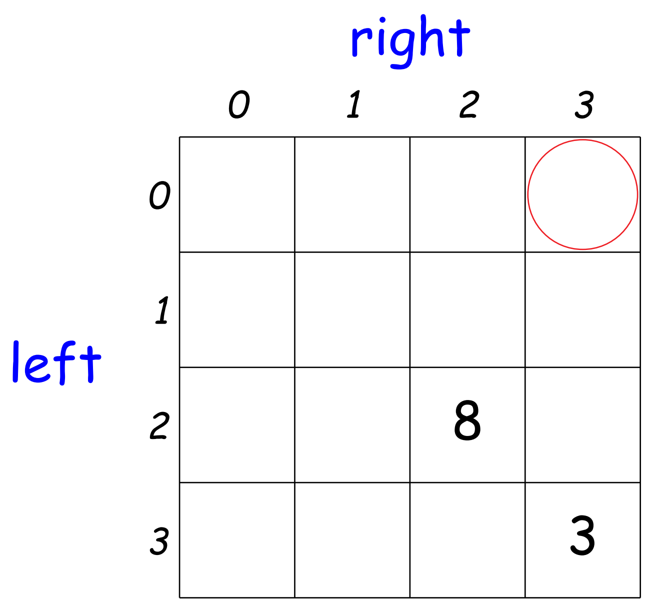

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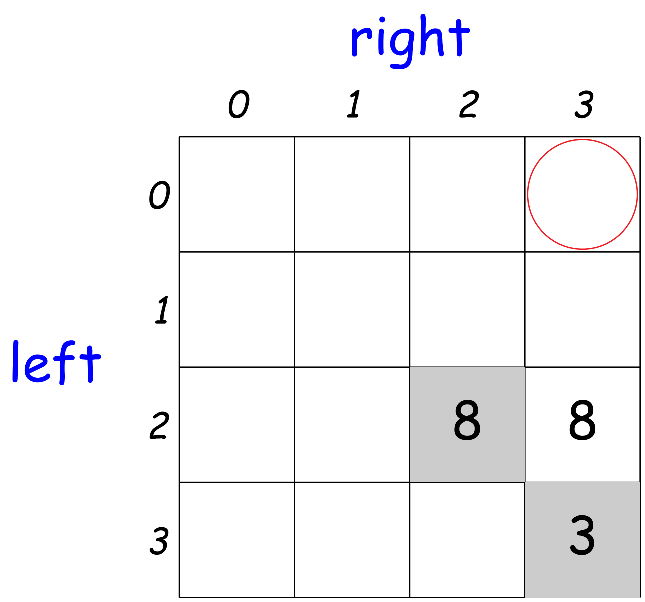

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- Suppose that we use as input `{ 2 1 8 3 }`. Our goal is to find the circled—(0,3)—entry in the memo table, which gets filled in like this (grayed boxes indicates table entries that are used to compute a new entry):

right

left	0	1	2	3
0				
1		1		
2			8	8
3				3

Iterative Version

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right

left	0	1	2	3
0				
1		1	8	
2			8	8
3				3

Iterative Version

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- Suppose that we use as input `{ 2 1 8 3 }`. Our goal is to find the circled—(0,3)—entry in the memo table, which gets filled in like this (grayed boxes indicates table entries that are used to compute a new entry):

right

left	0	1	2	3
0				
1		1	8	4
2			8	8
3				3

Iterative Version

- In the usual presentation of this idea—known as *dynamic programming*—we figure out ahead of time the order in which the memoized version will fill in `memo`, and write an explicit loop, saving the time needed to ceck whether the result has already been computed.
- Suppose that we use as input `{ 2 1 8 3 }`. Our goal is to find the circled—(0,3)—entry in the memo table, which gets filled in like this (grayed boxes indicates table entries that are used to compute a new entry):

right

left	0	1	2	3
0	2			
1		1	8	4
2			8	8
3				3

Iterative Version

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- Suppose that we use as input `{ 2 1 8 3 }`. Our goal is to find the circled—(0,3)—entry in the memo table, which gets filled in like this (grayed boxes indicates table entries that are used to compute a new entry):

right

left	*0*	*1*	*2*	*3*
0	2	2		
1		1	8	4
2			8	8
3				3

Iterative Version

- In the usual presentation of this idea—known as *dynamic programming*—we figure out ahead of time the order in which the memoized version will fill in `memo`, and write an explicit loop, saving the time needed to ceck whether the result has already been computed.
- Suppose that we use as input `{ 2 1 8 3 }`. Our goal is to find the circled—(0,3)—entry in the memo table, which gets filled in like this (grayed boxes indicates table entries that are used to compute a new entry):

right

left	0	1	2	3
0	2	2	9	
1		1	8	4
2			8	8
3				3

Iterative Version

- In the usual presentation of this idea—known as *dynamic programming*—we figure out ahead of time the order in which the memoized version will fill in `memo`, and write an explicit loop, saving the time needed to ceck whether the result has already been computed.
- Suppose that we use as input `{ 2 1 8 3 }`. Our goal is to find the circled—(0,3)—entry in the memo table, which gets filled in like this (grayed boxes indicates table entries that are used to compute a new entry):

right

left	0	1	2	3
0	2	2	9	10
1		1	8	4
2			8	8
3				3

The Iterative Program

- To fill in the table in this order, we can use the following:

```
int bestSum(int[] V) {
    int[][] memo = new int[V.length][V.length];
    int total;
    for (int left = V.length - 1; left >= 0; left -= 1) {
        memo[left][left] = total = V[left];
        for (int right = left + 1; right < V.length; right += 1) {
            total += V[right];
            int L = total - memo[left+1][right];
            int R = total - memo[left][right-1];
            memo[left][right] = Math.max(L, R);
        }
    }
    return memo[0][V.length-1];
}
```

- Generally, though, I say why bother unless the recursion would be too deep?

Compressing the Memo Table

- In fact, we can do even better, and use a one-dimentional memo table with only N entries.
- If you look at the previous animation of filling in the memo table, you'll see that we only ever reference entries immediately below and to the left of the item being computed.
- So we can actually just keep updating the same row.

```
int bestSum(int[] V) {
    int[] memo = new int[V.length];
    int total;
    for (int left = V.length - 1; left >= 0; left -= 1) {
        memo[left] = total = V[left];
        for (int right = left + 1; right < V.length; right += 1) {
            total += V[right];
            int L = total - memo[right];
            int R = total - memo[right-1];
            memo[right] = Math.max(L, R);
        }
    }
    return memo[V.length-1];
}
```

Longest Common Subsequence

- **Problem:** Find length of the longest string that is a subsequence of each of two other strings.
- **Example:** Longest common subsequence of
  "`sally␣sells␣sea␣shells␣by␣the␣seashore`" and
  "`sarah␣sold␣salt␣sellers␣at␣the␣salt␣mines`"
  is
  "`sa␣sl␣sa␣sells␣␣the␣sae`" (length 23)
- Similarity testing, for example.
- Obvious recursive algorithm:

```
/** Length of longest common subsequence of S0[0..k0-1]
 *  and S1[0..k1-1] (pseudo Java) */
static int lls(String S0, int k0, String S1, int k1) {
    if (k0 == 0 || k1 == 0) return 0;
    if (S0[k0-1] == S1[k1-1]) return 1 + lls(S0, k0-1, S1, k1-1);
    else return Math.max(lls(S0, k0-1, S1, k1), lls(S0, k0, S1, k1-1);
}
```

- Exponential, but obviously memoizable.

Memoized Longest Common Subsequence

```
/** Length of longest common subsequence of S0[0..k0-1] and S1[0..k1-1]. */
static int lls(String S0, int k0, String S1, int k1) {
    int[][] memo = new int[k0+1][k1+1];
    for (int[] row : memo) Arrays.fill(row, -1);
    return lls(S0, k0, S1, k1, memo);
}
private static int lls(String S0, int k0, String S1, int k1, int[][] memo) {
    if (k0 == 0 || k1 == 0) return 0;
    if (memo[k0][k1] == -1) {
        if (S0[k0-1] == S1[k1-1])
            memo[k0][k1] = 1 + lls(S0, k0-1, S1, k1-1, memo);
        else
            memo[k0][k1] = Math.max(lls(S0, k0-1, S1, k1, memo),
                                    lls(S0, k0, S1, k1-1, memo));
    }
    return memo[k0][k1];
}
```

Q: How fast will the memoized version be?

Memoized Longest Common Subsequence

```
/** Length of longest common subsequence of S0[0..k0-1] and S1[0..k1-1]. */
static int lls(String S0, int k0, String S1, int k1) {
    int[][] memo = new int[k0+1][k1+1];
    for (int[] row : memo) Arrays.fill(row, -1);
    return lls(S0, k0, S1, k1, memo);
}
private static int lls(String S0, int k0, String S1, int k1, int[][] memo) {
    if (k0 == 0 || k1 == 0) return 0;
    if (memo[k0][k1] == -1) {
        if (S0[k0-1] == S1[k1-1])
            memo[k0][k1] = 1 + lls(S0, k0-1, S1, k1-1, memo);
        else
            memo[k0][k1] = Math.max(lls(S0, k0-1, S1, k1, memo),
                                    lls(S0, k0, S1, k1-1, memo));
    }
    return memo[k0][k1];
}
```

Q: How fast will the memoized version be? $\Theta(k_0 \cdot k_1)$

Side Trip into Java: Enumeration Types

- Problem: Need a type with a few, named, discrete values.
- In the purest form, the only necessary operations are == and !=; the only property of a value of the type is that it differs from all others.
- In older versions of Java, used named integer constants:

```
interface Pieces {
  int BLACK_PIECE = 0,    // Fields in interfaces are static final.
      BLACK_KING = 1,
      WHITE_PIECE = 2,
      WHITE_KING = 3,
      EMPTY = 4;
}
```

- C and C++ provide *enumeration types* as a shorthand, with syntax like this:

```
enum Piece { BLACK_PIECE, BLACK_KING, WHITE_PIECE, WHITE_KING, EMPTY };
```

- But since all these values are basically **ints**, accidents can happen.

Enum Types in Java

- More modern versions of Java allow syntax like that of C or C++, but with more guarantees:

```
public enum Piece
     { BLACK_PIECE, BLACK_KING, WHITE_PIECE, WHITE_KING, EMPTY; }
```

- Defines `Piece` as a new reference type, a special kind of class type.
- The names `BLACK_PIECE`, etc., are static, final *enumeration constants* (or *enumerals*) of type `PIECE`.
- They are automatically initialized, and are the only values of the enumeration type that exist (cannot say **new** `Piece()`).
- Can safely use ==, and also `switch` statements:

```
boolean isKing(Piece p) {
    switch (p) {
    case BLACK_KING: case WHITE_KING: return true;
    default: return false;
    }
}
```

Making Enumerals Available Elsewhere

- Enumerals like `BLACK_PIECE` are static members of a class, not classes.
- Therefore, unlike *C* or *C++*, their declarations are not automatically visible outside the enumeration class definition.
- So, in other classes, must write `Piece.BLACK_PIECE`, which can get annoying.
- However, with version 1.5, Java has *static imports:* to import all static definitions of class `checkers.Piece` (including enumerals), you write

```
import static checkers.Piece.*;
```

  among the import clauses.
- Alas, *cannot* use this for enum classes in the anonymous package.

Operations on Enum Types

- Order of declaration of enumeration constants significant: `.ordinal()` gives the position (numbering from 0) of an enumeration value. Thus, `Piece.BLACK_KING.ordinal()` is 1.
- The array `Piece.values()` gives all the possible values of the type. Thus, you can write:

```
for (Piece p : Piece.values())
    System.out.printf("Piece value #%d is %s%n", p.ordinal(), p);
```

- The static function `Piece.valueOf` converts a String into a value of type `Piece`. So `Piece.valueOf("EMPTY") == EMPTY`.

Fancy Enum Types

- Enums are classes. You can define all the extra fields, methods, and constructors you want.
- Constructors are used only in creating enumeration constants. The constructor arguments follow the constant name:

```
enum Piece {
    BLACK_PIECE(BLACK, false, "b"), BLACK_KING(BLACK, true, "B"),
    WHITE_PIECE(WHITE, false, "w"), WHITE_KING(WHITE, true, "W"),
    EMPTY(null, false, " ");

    private final Side color;
    private final boolean isKing;
    private final String textName;

    Piece(Side color, boolean isKing, String textName) {
      this.color = color; this.isKing = isKing; this.textName = textName;
    }
    Side color() { return color; }
    boolean isKing() { return isKing; }
    String textName() { return textName; }
}
```