

CS61C Fall 2017 Discussion 9

Floating Point

The IEEE 754 standard defines a binary representation for floating point values using three fields:

- The *sign* determines the sign of the number (0 for positive, 1 for negative)
- The *exponent* is in **biased notation** with a bias of 127
- The *significand* is akin to unsigned, but used to store a fraction instead of an integer.

The below table shows the bit breakdown for the single precision (32-bit) representation:

Sign	Exponent	Significand
1 bit	8 bits	23 bits

There is also a double precision encoding format that uses 64 bits. This behaves the same as the single precision but uses 11 bits for the exponent (and thus a bias of 1023) and 52 bits for the significand.

How a float is interpreted depends on the values in the exponent and significand fields:

For normalized floats:

$$\text{Value} = (-1)^{\text{Sign}} \times 2^{(\text{Exponent} - \text{Bias})} \times 1.\text{significand}_2$$

For denormalized floats:

$$\text{Value} = (-1)^{\text{Sign}} \times 2^{(\text{Exponent} - \text{Bias} + 1)} \times 0.\text{significand}_2$$

Exponent	Significand	Meaning
0	Anything	Denorm
1-254	Anything	Normal
255	0	Infinity
255	Nonzero	NaN

Exercises

1. How many zeroes can be represented using a float? **2**
2. What is the largest finite positive value that can be stored using a single precision float?
 $0x7F7FFFFF = (2 - 2^{-23}) \times 2^{127}$
3. What is the smallest positive value that can be stored using a single precision float?
 $0x00000001 = 2^{-23} \times 2^{-126}$
4. What is the smallest positive normalized value that can be stored using a single precision float?
 $0x00800000 = 2^{-126}$

5. Convert the following numbers from binary to decimal or from decimal to binary:

0x00000000	8.25	0x00000F00	39.5625	0xFF94BEEF	$-\infty$
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$0x00000000 = 0$
 $8.25 = 0x41040000$
 $0x00000F00 = (2^{-12} + 2^{-13} + 2^{-14} + 2^{-15}) \times 2^{-126}$
 $39.5625 = 0x421E4000$
 $0xFF94BEEF = \text{NaN}$
 $-\infty = 0xFF800000$

AMAT

AMAT is the average (expected) time it takes for memory access. It can be calculated using this formula:

$$\text{AMAT} = \text{hit time} + \text{miss rate} \times \text{miss penalty}$$

Miss rates can be given in terms of either local miss rates or global miss rates. The *local miss rate* of a cache is the percentage of accesses into the particular cache that miss at the cache, while the *global miss rate* is the percentage of all accesses that miss at the cache.

Exercises

Suppose your system consists of:

- A L1\$ that hits in 2 cycles and has a local miss rate of 20%
- A L2\$ that hits in 15 cycles and has a global miss rate of 5%
- Main memory hits in 100 cycles

1. What is the local miss rate of L2\$?

$$\text{Local miss rate} = 5\% / 20\% = 0.25 = 25\%$$

2. What is the AMAT of the system?

$$\text{AMAT} = 2 + 20\% \times 15 + 5\% \times 100 = 10 \text{ (using global miss rates)}$$

$$\text{Alternatively, AMAT} = 2 + 20\% \times (15 + 25\% \times 100) = 10$$

3. Suppose we want to reduce the AMAT of the system to 8 or lower by adding in a L3\$. If the L3\$ has a local miss rate of 30%, what is the largest hit time that the L3\$ can have?

Let H = hit time of the cache. Using the AMAT equation, we can write:

$$2 + 20\% \times (15 + 25\% \times (H + 30\% \times 100)) \leq 8$$

Solving for H , we find that $H \leq 30$. So the largest hit time is 30 cycles.

Flynn Taxonomy

1. Explain SISD and give an example if available.

Single Instruction Single Data; each instruction is executed in order, acting on a single stream of data.

For example, traditional computer programs.

2. Explain SIMD and give an example if available.

Single Instruction Multiple Data; each instruction is executed in order, acting on multiple streams of data.

For example, the SSE Intrinsics.

3. Explain MISD and give an example if available.

Multiple Instruction Single Data; multiple instructions are executed simultaneously, acting on a single stream of data. There are no good modern examples.

4. Explain MIMD and give an example if available.

Multiple Instruction Multiple Data; multiple instructions are executed simultaneously, acting on multiple streams of data. For example, map reduce or multithreaded programs.