

Lecture 23 – Combinational Logic Blocks

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Long-Distance Wi-Fi ⇒

The difficulty of providing WiFi to remote municipalities may soon be over. Intel has figured out how to deliver signals at a low cost > 60 miles away. Since “you can’t lay cable”

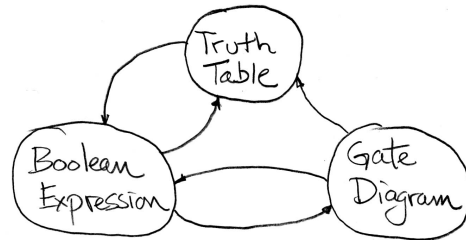


www.technologyreview.com/Infotech/20432/



Review

- Use this table and techniques we learned to transform from 1 to another

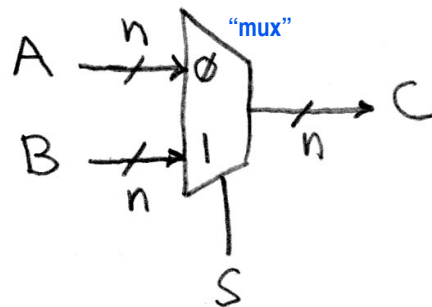


Today

- Data Multiplexors
- Arithmetic and Logic Unit
- Adder/Subtractor

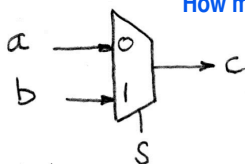


Data Multiplexor (here 2-to-1, n-bit-wide)



N instances of 1-bit-wide mux

How many rows in TT?

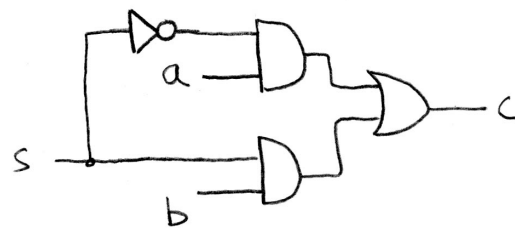


$$\begin{aligned}
 c &= \bar{s}a\bar{b} + \bar{s}ab + s\bar{a}b + sab \\
 &= \bar{s}(a\bar{b} + ab) + s(\bar{a}b + ab) \\
 &= \bar{s}(a(\bar{b} + b)) + s((\bar{a} + a)b) \\
 &= \bar{s}(a(1)) + s((1)b) \\
 &= \bar{s}a + sb
 \end{aligned}$$



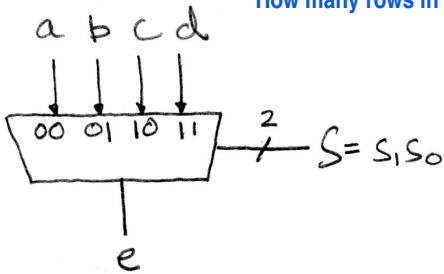
How do we build a 1-bit-wide mux?

$$\bar{s}a + sb$$



4-to-1 Multiplexor?

How many rows in TT?

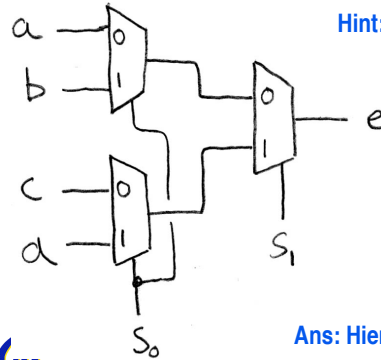


$$e = \bar{s}_1 \bar{s}_0 a + \bar{s}_1 s_0 b + s_1 \bar{s}_0 c + s_1 s_0 d$$



Is there any other way to do it?

Hint: NCAA tourney!



Ans: Hierarchically!



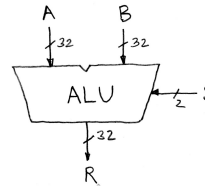
Administrivia

- Homework 5 due Friday (not tonight)
- All-hands EECS Faculty mini-retreat on Friday
 - TA will cover



Arithmetic and Logic Unit

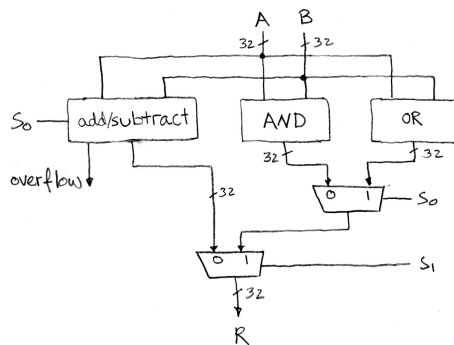
- Most processors contain a special logic block called "Arithmetic and Logic Unit" (ALU)
- We'll show you an easy one that does ADD, SUB, bitwise AND, bitwise OR



when $S=00$, $R=A+B$
 when $S=01$, $R=A-B$
 when $S=10$, $R=A \text{ AND } B$
 when $S=11$, $R=A \text{ OR } B$



Our simple ALU



Adder/Subtractor Design -- how?

- Truth-table, then determine canonical form, then minimize and implement as we've seen before
- Look at breaking the problem down into smaller pieces that we can cascade or hierarchically layer



Adder/Subtractor – One-bit adder LSB...

| | | | | |
|---|-------|-------|-------|-------|
| | a_3 | a_2 | a_1 | a_0 |
| + | b_3 | b_2 | b_1 | b_0 |
| | s_3 | s_2 | s_1 | s_0 |

| | | | |
|-------|-------|-------|-------|
| a_0 | b_0 | s_0 | c_1 |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

$s_0 =$
 $c_1 =$



Adder/Subtractor – One-bit adder (1/2)...

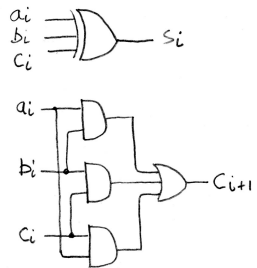
| | | | | |
|---|-------|-------|-------|-------|
| | a_3 | a_2 | a_1 | a_0 |
| + | b_3 | b_2 | b_1 | b_0 |
| | s_3 | s_2 | s_1 | s_0 |

| | | | | |
|-------|-------|-------|-------|-----------|
| a_i | b_i | c_i | s_i | c_{i+1} |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$s_i =$
 $c_{i+1} =$



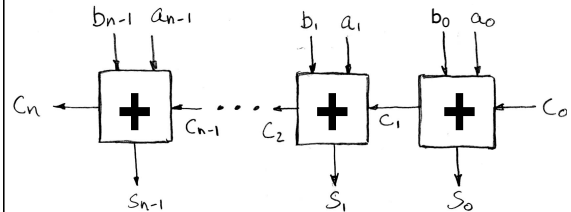
Adder/Subtractor – One-bit adder (2/2)...



$s_i = \text{XOR}(a_i, b_i, c_i)$
 $c_{i+1} = \text{MAJ}(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$



N 1-bit adders \Rightarrow 1 N-bit adder



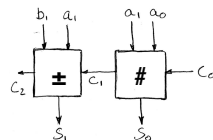
What about overflow?
Overflow = c_n ?



What about overflow?

Consider a 2-bit signed # & overflow:

- 10 = -2 + -2 or -1
- 11 = -1 + -2 only
- 00 = 0 NOTHING!
- 01 = 1 + 1 only



Highest adder

- $C_1 = \text{Carry-in} = C_{in}$, $C_2 = \text{Carry-out} = C_{out}$
- No C_{out} or $C_{in} \Rightarrow$ NO overflow!
- C_{in} and $C_{out} \Rightarrow$ NO overflow!

What op?

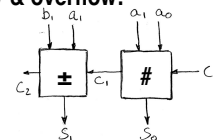
- C_{in} , but no $C_{out} \Rightarrow$ A,B both > 0, overflow!
- C_{out} , but no $C_{in} \Rightarrow$ A,B both < 0, overflow!



What about overflow?

Consider a 2-bit signed # & overflow:

- 10 = -2
- 11 = -1
- 00 = 0
- 01 = 1



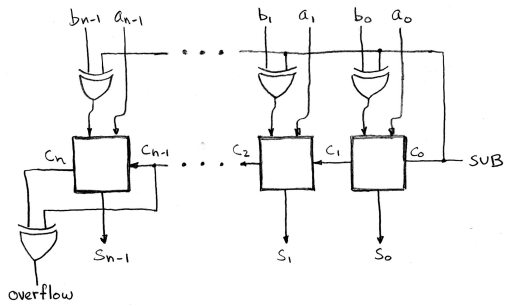
Overflows when...

- C_{in} , but no $C_{out} \Rightarrow$ A,B both > 0, overflow!
- C_{out} , but no $C_{in} \Rightarrow$ A,B both < 0, overflow!

overflow = $c_n \text{ XOR } c_{n-1}$



Extremely Clever Subtractor



Peer Instruction

- Truth table for mux with 4-bits of signals has 2^4 rows
- We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl
- If 1-bit adder delay is T, the N-bit adder delay would also be T

| | ABC |
|----|-----|
| 0: | FFF |
| 1: | FFT |
| 2: | FTF |
| 3: | FTT |
| 4: | TFF |
| 5: | TFT |
| 6: | FTT |
| 7: | TTT |



Peer Instruction Answer

"And In conclusion..."

- Use muxes to select among input
 - S input bits selects 2^S inputs
 - Each input can be n-bits wide, indep of S
- Can implement muxes hierarchically
- ALU can be implemented using a mux
 - Coupled with basic block elements
- N-bit adder-subtractor done using N 1-bit adders with XOR gates on input
 - XOR serves as conditional inverter

