CS61c Spring 2014 Discussion 0 – Number Representation

1 Unsigned Integers

By now we should all be somewhat comfortable with non-decimal bases. As a reminder, if we have an n-digit unsigned numeral $d_{n-1}d_{n-2}\ldots d_0$ in radix (or base) r, then the value of that numeral is $\sum_{i=0}^{n-1} r^i d_i$, which is just fancy notation to say that instead of a 10's or 100's place we have an r's or r^2's place. For binary, decimal, and hex we just let r be 2, 10, and 16, respectively.

Recall also that we often have cause to write down unreasonably large numbers, and our preferred tool for doing that is the IEC prefixing system: Ki = 2^{10}, Mi = 2^{20}, Gi = 2^{30}, Ti = 2^{40}, Pi = 2^{50}, Ei = 2^{60}, Zi = 2^{70}, Yi = 2^{80}.

1.1 We don't have calculators during exams, so let's try this by hand

1. Convert the following numbers from their initial radix into the other two common radices: 0b10010011, 0xD3AD, 63, 0b00100100, 0xB33F, 0, 39, 0x7EC4, 437

2. Write the following numbers using IEC prefixes: 2^{16}, 2^{34}, 2^{27}, 2^{61}, 2^{43}, 2^{47}, 2^{36}, 2^{58}.

3. Write the following numbers as powers of 2: 2 Ki, 256 Pi, 512 Ki, 64 Gi, 16 Mi, 128 Ei

2 Signed Integers

Unsigned binary numbers work to store natural numbers, but many calculations use negative numbers as well. To deal with this a number of different schemes have been used to represent signed numbers.

2.1 Sign and Magnitude and One's complement

Both of these schemes are relatively simple conceptually, but have been replaced by cleverer representations. Why?

- Most significant bit tells you the sign: 1 if negative, 0 if positive.
- Positive values can be treated just like unsigned integers.
- To invert the sign of a sign and magnitude number flip the MSB.
- To invert the sign of a one's complement number flip all the bits.

2.2 Biased Notation

- Like an unsigned int, but offset by $-(2^{n-1} - 1)$, where n is the number of bits in the numeral. Aside: Technically we could choose any bias we please, but the choice presented here is extraordinarily common.
- Formally, if we have an n-bit biased notation number with bits $d_{n-1}d_{n-2}\ldots d_0$, then the value of the numeral is $-(2^{n-1} - 1) + \sum_{i=0}^{n-1} 2^i d_i$.
- Just one zero, but it's not at 0b0.
- Addition is a little weird, but not overwhelmingly so.

2.3 Two's complement

- Two's complement is the standard solution for representing signed integers.
  - Most significant bit has a negative value, all others have positive.
  - Otherwise exactly the same as unsigned integers.
- A neat trick for flipping the sign of a two's complement number: flip all the bits and add 1.
- Addition is exactly the same as with an unsigned number.
- Only one 0, and it's located at 0b0.

2.4 Exercises

For the following questions assume an 8 bit integer. Answer each question for the case of a sign and magnitude number, a one's complement number, a biased notation number, and a two's complement number.

1. What is the largest integer? The largest integer + 1?
2. How do you represent the numbers 0, 1, and -1?
3. How do you represent 17, -17?
4. What is the largest integer that can be represented by *any* encoding scheme that only uses 8 bits?
5. Prove that the two's complement inversion trick is valid (i.e. that x and $\overline{x} + 1$ sum to 0).
6. Explain where each of the three radices shines and why it is preferred over other bases in a given context.

3 Counting

Bitstrings can be used to represent more than just numbers. In fact, we use bitstrings to represent *everything* inside a computer. And, because we don't want to be wasteful with bits it is important that to remember that n bits can be used to represent 2^n distinct things. To reiterate, n bits can represent up to 2^n distinct objects.

3.1 Exercises

1. If the value of a variable is $0, \pi$ or e, what is the minimum number of bits needed to represent it.
2. If we need to address 3 TiB of memory and we want to address every byte of memory, how long does an address need to be?
3. If the only value a variable can take on is e, how many bits are needed to represent it.