

1 Understanding T/I/O

When working with caches, we have to be able to break down the memory addresses we work with to understand where they fit into our caches. There are three fields:

- Tag** - Used to distinguish different blocks that use the same index - Number of bits: leftovers
- Index** - The set that this piece of memory will be placed in - Number of bits: $\log_2(\# \text{ of indices})$
- Offset** - The location of the byte in the block - Number of bits: $\log_2(\text{size of block})$

Given these definitions, the following is true:

$$\log_2(\text{memory size}) = \text{address bit-width} = \# \text{ tag bits} + \# \text{ index bits} + \# \text{ offset bits}$$

Another useful equality to remember is:

$$\text{cache size} = \text{block size} * \text{num blocks}$$

- 1.1 Assume we have a direct-mapped byte-addressed cache with capacity 32B and block size of 8B. Of the 32 bits in each address, which bits do we use to find the index of the cache to use?

We use bits 3 and 4, where we denote the MSB as 31 and the LSB as 0.

- 1.2 Which bits are our tag bits? What about our offset?

The offset is 3 bits, and our tag is the remaining high-order bits.

- 1.3 Classify each of the following byte memory accesses as a cache hit (H), cache miss (M), or cache miss with replacement(R). It is probably best to try drawing out the cache before going through so that you can have an easier time seeing the replacements in the cache. The following white space is to do this:

Address	T/I/O	Hit, Miss, Replace
0x00000004		
0x00000005		
0x00000068		
0x000000C8		
0x00000068		
0x000000DD		
0x00000045		
0x00000004		
0x000000C8		

- 0x00000004 Index 0, Tag 0: M
- 0x00000005 Index 0, Tag 0: H
- 0x00000068 Index 1, Tag 3: M
- 0x000000C8 Index 1, Tag 6: R
- 0x00000068 Index 1, Tag 3: R
- 0x000000DD Index 3, Tag 6: M
- 0x00000045 Index 0, Tag 2: R
- 0x00000004 Index 0, Tag 0: R
- 0x000000C8 Index 1, Tag 6: R

Note that the M and R distinction here is for student understanding, and that the cache doesn't behave differently for these cases.

2 The 3 C's of Misses

2.1 Classify each M and R above as one of the 3 types of misses described below:

1. Compulsory: First time you ask the cache for a certain block. A miss that must occur when you first bring in a block. Reduce compulsory misses by having a longer cache lines (bigger blocks), which bring in the surrounding addresses along with our requested data. Can also pre-fetch blocks beforehand using a hardware prefetcher (a special circuit that tries to guess the next few blocks that you will want).
2. Conflict: Occurs if, hypothetically, you went through the ENTIRE string of accesses with a fully associative cache and wouldn't have missed for that specific access. Increasing the associativity or improving the replacement policy would remove the miss.
3. Capacity: The only way to remove the miss is to increase the cache capacity, as even with a fully associative cache, we had to kick a block out at some point.

Note: There are many different ways of fixing misses. The name of the miss doesn't necessarily tell us the best way to reduce the number of misses.

- 0x00000004, Compulsory
- 0x00000005, N/A
- 0x00000068, Compulsory
- 0x000000C8, Compulsory
- 0x00000068, Conflict
- 0x000000DD, Compulsory
- 0x00000045, Compulsory

- 0x00000004, Capacity
- 0x000000C8, Capacity

3 Code Analysis

Given the follow chunk of code, analyze the hit rate given that we have a byte-addressed computer with a total memory of **1 MiB**. It also features a **16 KiB** Direct-Mapped cache with **1 KiB** blocks. Assume that your cache begins cold.

```
#define NUM_INTS 8192 // 2^13
int A[NUM_INTS]; // A lives at 0x10000
int i, total = 0;
for (i = 0; i < NUM_INTS; i += 128) {
    A[i] = i; // Line 1
}
for (i = 0; i < NUM_INTS; i += 128) {
    total += A[i]; // Line 2
}
```

3.1 How many bits make up a memory address on this computer?

We take $\log_2(1 \text{ MiB}) = \log_2(2^{20}) = 20$

3.2 What is the T:I:O breakdown?

$$\text{Offset} = \log_2(1 \text{ KiB}) = \log_2(2^{10}) = 10$$

$$\text{Index} = \log_2(\frac{16 \text{ KiB}}{1 \text{ KiB}}) = \log_2(16) = 4$$

$$\text{Tag} = 20 - 4 - 10 = 6$$

3.3 Calculate the cache hit rate for the line marked Line 1:

The integer accesses are $4 * 128 = 512$ bytes apart, which means there are 2 accesses per block. The first access in each block is a compulsory cache miss, but the second is a hit because $A[i]$ and $A[i+128]$ are in the same cache block. Resulting in a hit rate of **50%**.

3.4 Calculate the cache hit rate for the line marked Line 2:

The size of A is $8192 * 4 = 2^{15}$ bytes. This is exactly twice the size of our cache. At the end of Line 1, we have the second half of A inside our cache, but Line 2 starts with the first half of A. Thus, we cannot reuse any of the cache data brought in from Line 1 and must start from the beginning. Thus our hit rate is the same as Line 1 since we access memory in the same exact way as Line 1. We don't have to consider cache hits for total, as the compiler will most likely store it in a register. Resulting in a hit rate of **50%**.

4 Cache Associativity

In the previous problems, we have a Direct-Mapped cache, in which blocks map to specifically one slot in our cache. This is good for quick replacement and finding out

block, but not good for efficiency of space!

This is where we bring associativity into the matter. We define associativity as the number of slots a block can potentially map to in our cache. Thus, a Fully-Associative cache has the most associativity, meaning every block can go anywhere in the cache.

For an N -way associative cache, the following is true:

$$N * \# \text{ sets} = \# \text{ blocks}$$

- 4.1 Heres some practice involving a 2-way set associative cache. This time we have an 8-bit address space, 8 B blocks, and a cache size of 32 B. Classify each of the following accesses as a cache hit (H), cache miss (M) or cache miss with replacement (R). For any misses, list out which type of miss it is.

Address	T/I/O	Hit, Miss, Replace
0b0000 0100		
0b0000 0101		
0b0110 1000		
0b1100 1000		
0b0110 1000		
0b1101 1101		
0b0100 0101		
0b0000 0100		
0b1100 1000		

For this solution, we assume that we have an LRU replacement policy. In general, this is not necessarily the case.

0b0000 0100	Tag 0000, Index 0, Offset 100 - M, Compulsory
0b0000 0101	Tag 0000, Index 0, Offset 101 - H
0b0110 1000	Tag 0110, Index 1, Offset 000 - M, Compulsory
0b1100 1000	Tag 1100, Index 1, Offset 000 - M, Compulsory
0b0110 1000	Tag 0110, Index 1, Offset 000 - H
0b1101 1101	Tag 1101, Index 1, Offset 101 - R, Compulsory
0b0100 0101	Tag 0100, Index 0, Offset 101 - M, Compulsory
0b0000 0100	Tag 0000, Index 0, Offset 100 - H
0b1100 1000	Tag 1100, Index 1, Offset 000 - R, Capacity

- 4.2 What is the hit rate of our above accesses?

$$\frac{3 \text{ hits}}{9 \text{ accesses}} = \frac{1}{3} \text{ hit rate}$$