CS61c Summer 2014 Discussion 1 – Number Representation

1 Unsigned Integers

By now we should all be somewhat comfortable with non-decimal bases. As a reminder, if we have an n-digit unsigned numeral $d_{n-1}d_{n-2}\ldots d_0$ in radix (or base) r, then the value of that numeral is $\sum_{i=0}^{n-1} r^i d_i$, which is just fancy notation to say that instead of a 10's or 100's place we have an r's or r^2's place. For binary, decimal, and hex we just let r be 2, 10, and 16, respectively. Recall also that we often have cause to write down unreasonably large numbers, and our preferred tool for doing that is the IEC prefixing system:

Ki	Mi	Gi	Ti	Pi	Ei	Zi	Yi
2^{10}	2^{20}	2^{30}	2^{40}	2^{50}	2^{60}	2^{70}	2^{80}

1.1 We don't have calculators during exams, so let's try this by hand

1. Convert the following numbers from their initial radix into the other two common radices: 0b10010011, 0xD3AD, 63, 0b00100100, 0xB33F, 0, 39, 0x7EC4, 437

2. Write the following numbers using IEC prefixes: 2^{16}, 2^{34}, 2^{27}, 2^{61}, 2^{43}, 2^{47}, 2^{36}, 2^{58}

3. Write the following numbers as powers of 2: 2 Ki, 256 Pi, 512 Ki, 64 Gi, 16 Mi, 128 Ei

2 Signed Integers

Unsigned binary numbers work to store natural numbers, but many calculations use negative numbers as well. To deal with this, a number of different schemes have been used to represent signed numbers.

2.1 Sign and Magnitude and One's complement

- Most significant bit tells you the sign: 1 if negative, 0 if positive.
- Positive values can be treated just like unsigned integers.
- To invert the sign of a sign and magnitude number, flip the MSB.
- To invert the sign of a one's complement number, flip all the bits.

Both of these schemes are relatively simple conceptually, but have been replaced by cleverer representations. Why?

2.2 Biased Notation

- Like an unsigned int, but offset by $-(2^{n-1}-1)$, where n is the number of bits in the numeral. Aside: Technically we could choose any bias we please, but the choice presented here is highly common.
- Formally, if we have an n-bit biased notation number with bits $d_{n-1}d_{n-2}\ldots d_0$, then the value of the numeral is $-(2^{n-1}-1)+\sum_{i=0}^{n-1} 2^i d_i$.
- Just one zero, but it's not at 0b0.
- Addition is a little weird, but not overwhelmingly so.

2.3 Two's complement

- Two's complement is the standard solution for representing signed integers.
  - Most significant bit has a negative value, all others have positive values.
  - Otherwise exactly the same as unsigned integers.
- A neat trick for flipping the sign of a two's complement number: flip all the bits and add 1.
- Addition is exactly the same as with an unsigned number.
- Only one 0, and it's located at 0b0.

2.4 Exercises

For the following questions assume an 8 bit integer. Answer each question for the case of a sign and magnitude number, a one's complement number, a biased notation number (using the bias calculation from before), and a two's complement number.

1. What is the largest integer? The largest integer + 1?
2. How do you represent the numbers 0, 1, and -1?
3. How do you represent 17, -17?
4. What is the largest integer that can be represented by *any* encoding scheme that only uses 8 bits?
5. Prove that the two's complement inversion trick is valid (i.e. that x and $\overline{x} + 1$ sum to 0).
6. Explain where each of the three radices shines and why it is preferred over other bases in a given context.

3 Counting

Bitstrings can be used to represent more than just numbers. In fact, we use bitstrings to represent *everything* inside a computer. And, because we don't want to be wasteful with bits it is important that to remember that n bits can be used to represent 2^n distinct things. To reiterate, n bits can represent up to 2^n distinct objects.

3.1 Exercises

1. If the value of a variable is $0, \pi$ or e, what is the minimum number of bits needed to represent it?
2. If we need to address 3 TiB of memory and we want to address every byte of memory, how long does an address need to be?
3. If the only value a variable can take on is e, how many bits are needed to represent it?