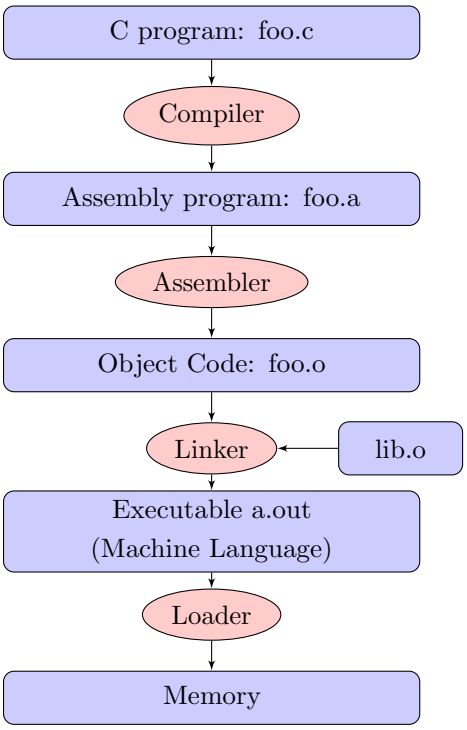
CS 61C Summer 2019

CALL, Floating Point

Discussion 4: June 10, 2019

1 CALL

The following is a diagram of the CALL stack detailing how C programs are built and executed by machines:



1.1 What is the Stored Program concept and what does it enable us to do?

It is the idea that instructions are just the same as data, and we can treat them as such. This enables us to write programs that can manipulate other programs!

1.2 How many passes through the code does the Assembler have to make? Why?

Two, one to find all the label addresses and another to convert all instructions while resolving any forward references using the collected label addresses.

1.3 Describe the six main parts of the object files outputed by the Assembler (Header, Text, Data, Relocation Table, Symbol Table, Debugging Information).

- Header: Size and position of other parts
- Text: The machine code
- Data: Binary representation of any data in the source file

- Relocation Table: Identifies lines of code that need to be "handled" by Linker (jumps to external labels (e.g. lib files), reference to static data)
- Symbol Table: List of file labels and data that can be referenced across files
- Debugging Information: Additional information for debuggers

1.4 Which step in CALL resolves relative addressing? Absolute addressing?

Assembler, linker

1.5 What does RISC stand for? How is this related to pseudoinstructions?

Reduced Instruction Set Computing. Minimal set of instructions leads to many lines of code. Pseudoinstructions are more complex instructions intended to make assembly programming easier for the coder. These are converted to basic instructions by the assembler.

2 Floating Point

The IEEE 754 standard defines a binary representation for floating point values using three fields.

- The *sign* determines the sign of the number (0 for positive, 1 for negative).
- The *exponent* is in **biased notation**. For instance, the bias is 127 for single-precision floating point numbers.
- The *significand* or *mantissa* is akin to unsigned integers, but used to store a fraction instead of an integer.

The below table shows the bit breakdown for the single precision (32-bit) representation. The leftmost bit is the MSB and the rightmost bit is the LSB.

1	8	23
Sign	Exponent	Mantissa/Significand/Fraction

For normalized floats:
Value $= (-1)^{Sign} * 2^{Exp-Bias} * 1.\textbf{significand}_2$
For denormalized floats:
Value $= (-1)^{Sign} * 2^{Exp-Bias+1} * 0.\textbf{significand}_2$

Exponent	**Significand**	**Meaning**
0	Anything	Denorm
1-254	Anything	Normal
255	0	Infinity
255	Nonzero	NaN

Note that in the above table, our exponent has values from 0 to 255. When translating between binary and decimal floating point values, we must remember that there is a bias for the exponent.

2.1 How many zeroes can be represented using a float?

2

2.2 What is the largest finite positive value that can be stored using a single precision float?

`0x7F7FFFFF` $= (1 + (1 - 2^{-23})) * 2^{127}$

The mantissa for the largest value will be 23 1's. This corresponds to a value of

$$.11\ldots1 = 2^{-1} + 2^{-2} + \cdots + 2^{-23} = 2^{-23}(2^{22} + 2^{21} + \cdots + 1)$$

Here, we apply the formula that $\sum_{i=0}^{n-1} 2^i = 2^n - 1$, so we have that the mantissa is

$$2^{-23}(2^{22} + 2^{21} + \cdots + 1) = 2^{-23}(2^{23} - 1) = 1 - 2^{-23}$$

We have $1 + (1 - 2^{-23})$ since we this is a normalized number and thus has a 1 to the left of the decimal point.

2.3 What is the smallest positive value that can be stored using a single precision float?

`0x00000001` $= 2^{-23} * 2^{-126}$

2.4 What is the smallest positive normalized value that can be stored using a single precision float?

`0x00800000` $= 2^{-126}$

2.5 Cover the following single-precision floating point numbers from binary to decimal or from decimal to binary. You may leave your answer as an expression.

- 0x00000000

  0

- 8.25

  0x41040000

- 0x00000F00

  $(2^{-12} + 2^{-13} + 2^{-14} + 2^{-15}) * 2^{-126}$

- 39.5625

  0x421E4000

- 0xFF94BEEF

  NaN

- -∞

  0xFF800000

3 More Floating Point Representation

Not every number can be represented perfectly using floating point. For example, $\frac{1}{3}$ can only be approximated and thus must be rounded in any attempt to represent it. For this question, we will only look at positive numbers.

3.1 What is the next smallest number larger than 2 that can be represented completely?

For this question, you increment the number by the smallest amount possible. This is the same as incrementing the significand by 1 at the rightmost location.

$(1 + 2^{-23}) * 2 = 2 + 2^{-22}$

3.2 What is the next smallest number larger than 4 that can be represented completely?

For this question, you increment the number by the smallest amount possible. This is the same as incrementing the significand by 1 at the rightmost location.
$(1 + 2^{-23}) * 4 = 4 + 2^{-21}$

3.3 Define stepsize to be the distance between some value x and the smallest value larger than x that can be completely represented. What is the step size for 2? 4?

This would be the amount added in part 1. This gives 2^{-22} and 2^{-21}.

3.4 Now let's see if we can generalize the stepsize for normalized numbers (we can do so for denorms as well, but we won't in this question). If we are given a normalized number that is not the largest representable normalized number with exponent value x and with significand value y, what is the stepsize at that value? Hint: There are 23 significand bits.

Here we need to generalize the solution we got in 1 and 2. However, this is the same approach just increment the signifcand by the 1.
$curr\_number = 2^{x-127} + 2^{x-127} * y$
$next\_number = 2^{x-127} + 2^{x-127} * y + 2^{x-127} * 2^{-23}$
$stepsize = next\_number - curr\_number = 2^{x-150}$

3.5 Now let's apply this technique. What is the largest odd number that we can represent? Part 3 should be very useful in finding this answer.

To find the largest odd number we can represent, we want to find when odd numbers will stop appearing. This will be with step size of 2.

As a result, plugging into 3: $2 = 2^{x-150} \rightarrow x = 151$
This means the number before $2^{151-127}$ was a distance of 1 (it is the first value whose stepsize is 2) and no number after will be odd. Thus, the odd number is simply subtracting the previous step size of 1.This gives,
$2^{24} - 1$