LECTURE #25

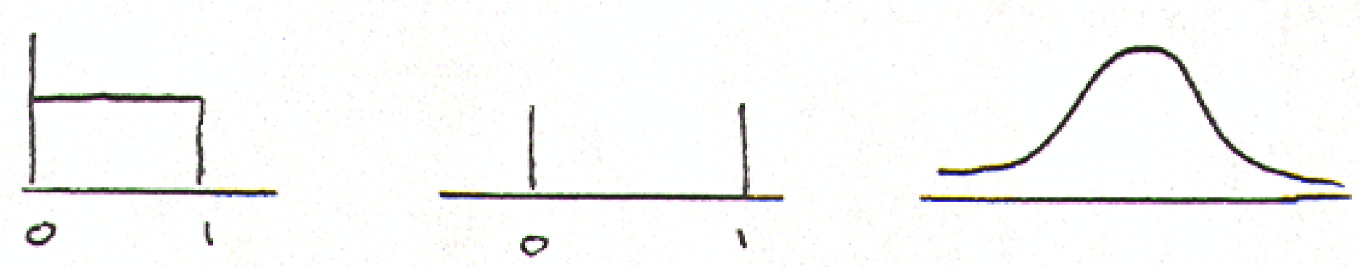
STATISTICS AND THE NORMAL DISTRIBUTION.

LAW OF LARGE NUMBERS

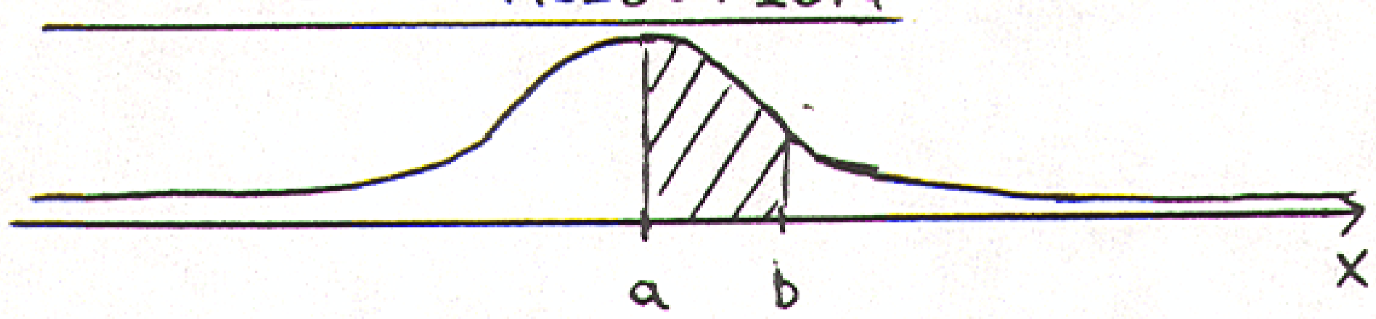
LET $X_1, \cdots, X_n$ BE iid (INDEPENDENTLY IDENTICALLY DISTRIBUTED) r.v. WITH EXPECTATION $\mu = E(X_i)$ SAMPLE MEAN $\bar{m} = \frac{1}{n}\sum_i X_i$ THEN FOR ANY $\alpha > 0$ $Pr[|\bar{m} - \mu| \geq \alpha] \to 0$ AS $n \to \infty$

$[|E(A_n) - E(X_i)|]$

$A_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ X_i:



NORMAL DISTRIBUTION

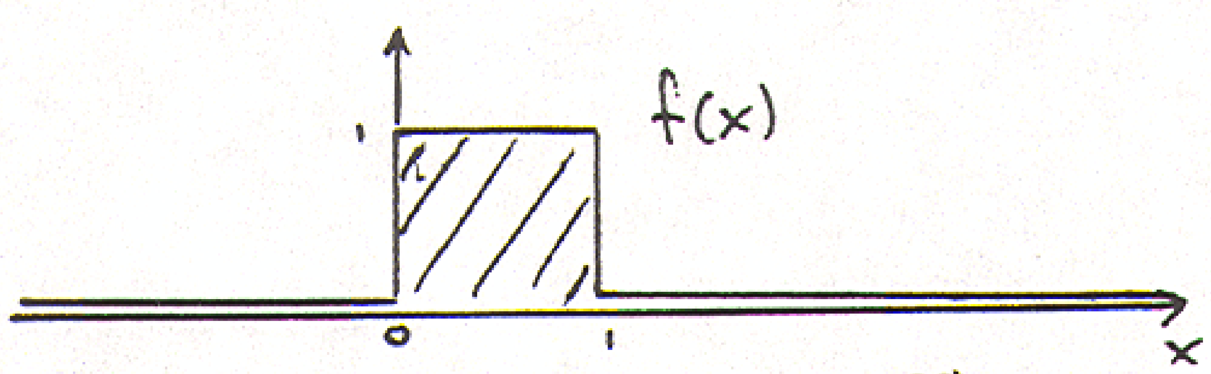


REAL VALUED RANDOM VARIABLES.

$f(x)$ = DENSITY FUNCTION

$Pr[a \leq x \leq b]$

DEFINITION:> REAL VALUED r.v. X, A REAL VALUED FUNCTION $f(x)$ IS A (PROBABILITY) DENSITY FUNCTION FOR X IF $Pr[x \leq a] = \int_{-\infty}^{a} f(x)\,dx$



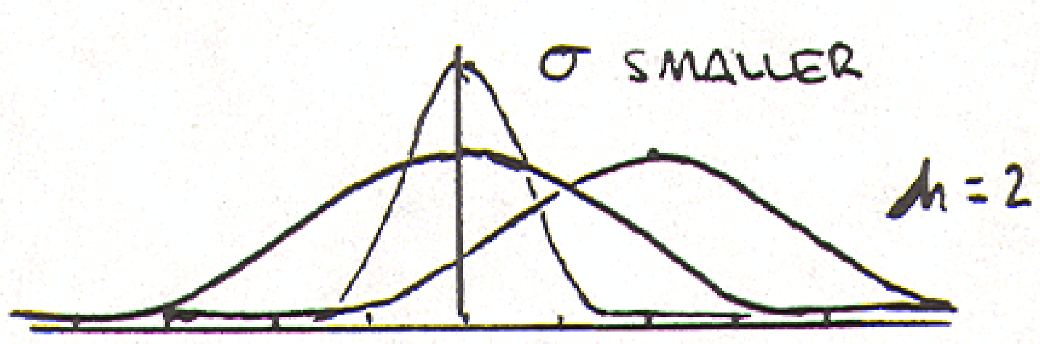
UNIFORM DISTRIBUTION ON [0, 1]

$E(x) = \frac{1}{2}$ $Var(x) = \frac{1}{12}$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$ $\int_{-\infty}^{\infty} f(x)\,dx = 1$ $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$ $Var(x) = E(x^2) - E(x)^2$

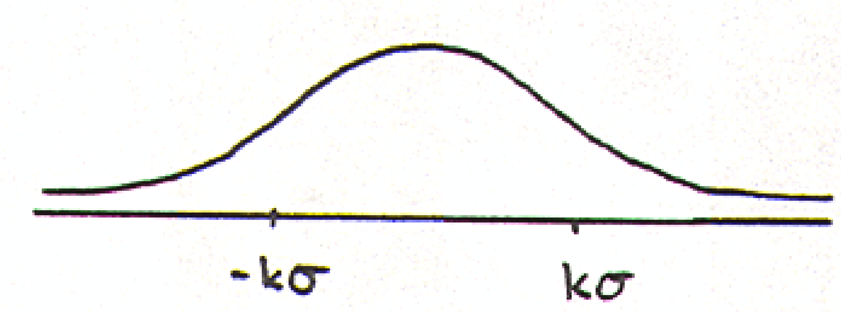
DEFINITION:> (NORMAL DISTRIBUTION) THE NORMAL DISTRIBUTION WITH MEAN μ AND VARIANCE σ^2 IS THE DISTRIBUTION WITH DENSITY FOR:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \qquad \left.\begin{matrix}\mu = 0\\ \sigma^2 = 1\end{matrix}\right\} N(\mu, \sigma^2),\ N(0,1) \longleftrightarrow f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



CENTRAL LIMIT THEOREM: LET $X_1, \cdots, X_n$ BE i.i.d. r.v. $E(X_i) = \mu$ AND $Var(X_i) = \sigma^2$. $A_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ AS $n \to \infty$ DISTRIBUTION OF THE A_n APPROACHES $N(\mu, \frac{\sigma^2}{n})$ ie. NORMAL DISTRIBUTION WITH MEAN μ AND VARIANCE $\frac{\sigma^2}{n}$

$N(\mu, \sigma^2)$
1) SYMMETRIC ABOUT μ.
2) BEYOND $\pm 3.5\sigma$ IS ESSENTIALLY 0.
3) STEEPER/HIGHER AS σ SHRINKS.
4) $[-\sigma, +\sigma]$ 68.27% $\quad$ 95% $\to [-1.96\sigma, +1.96\sigma]$ SIGNIFICANT
5) $[-2\sigma, +2\sigma]$ 95.45% $\quad$ 99% $\to [-2.58\sigma, +2.58\sigma]$ VERY SIGNIFICANT
6) $[-3\sigma, +3\sigma]$ 99.97%



$X_1, \cdots, X_n$ iid SAMPLE MEAN $\bar{m} = \frac{X_1 + \cdots + X_n}{n}$ SAMPLE VARIANCE $\frac{(X_1 - \bar{m})^2 + \cdots + (X_n - \bar{m})^2}{n-1} \approx \frac{(X_1 - m)^2 + \cdots + (X_{n-}}{n}$