

## Counting

### Counting Committees:

1. Let's say you have a company with  $n$  employees, and you would like to form a committee of  $k$  employees. Each employee in the committee is designated to perform one of  $k$  different tasks.
  - (a) If each employee can only perform one task at a time, how committee choices do you have?
  - (b) If each employee can perform up to  $k$  tasks at a time, how many choices do you have?
2. Now assume you don't have designated tasks; you simply want to form a committee of  $k$  employees who will work together on one task. How many different committees are possible?
  - (a) If the work is divided evenly between the  $k$  chosen employees (so you actually need  $k$  committee members), how many different committees are possible?
  - (b) **Challenge:** What if instead of needing  $k$  employees, you want  $k$  hours of committee work done, and employees sign up to be part of the committee by signing up for some number of 1-hour work increments (notice it's possible for only one person to complete all  $k$  hours of the committee work). In this setting, how many ways can you choose the committee?

### Counting and Probability:

1. Go back to the example above in which you have  $n$  employees, and you want to form a committee of size  $k$  in which each committee member does a different job. Say each employee can only perform one task at a time.
  - (a) Say that one of the jobs is being committee chair. Suppose you have one most trusted employee,  $X$ . How many committee choices do you have if you always designate  $X$  as the chair?
  - (b) If you choose a random committee, what is the probability that  $X$  is the committee chair?
2. Now, say you want a committee with  $k$  jobs, and each person can do more than one job.
  - (a) Say that you always make sure  $X$  is the committee chair, and that he also has the job of note-taker. How many committee choices do you have if you always give both of these jobs to  $X$ ?
  - (b) If you choose committees completely at random, what is the probability that  $X$  is chair and note-taker of the committee?
3. Finally, say you want a committee in which all members do an equal amount of work.
  - (a) Now, how many committee choices do you have if you always make sure  $X$  is part of the committee?
  - (b) If you choose a random committee, what is the probability that  $X$  is part of the committee?