1. **More Proofs!**

A combinatorial proof is a proof which shows that two quantities are the same by explaining that each quantity is a different way of counting the same thing. This question is intended to help you see how this technique is applied.

Select all choices that are valid ways of counting the number of squares in an $n \times n$ grid.

(a) In an $n \times n$ grid, there are n rows of squares, each of which has n squares in it. Thus, there are n^2 squares in an $n \times n$ grid.

(b) We know there are exactly n squares on the diagonal. Now, when we remove the diagonal, we have two equally sized triangles that have $n-1$ squares on the hypotenuse. When we remove those, we end up with smaller triangles with $n-2$ squares on the hypothenuse. We continue this until we are left with one square on each side, and we've counted all of the squares in the grid. This gives us a total of $n + 2\sum_{k=1}^{n-1} k$ squares in the grid.

(c) Take the $(n-1) \times (n-1)$ subgrid that is the upper lefthand corner of this grid. This subgrid has $n-1$ rows, each of which has $n-1$ squares, so this part contributes $(n-1)^2$ squares. Now, the squares that we excluded from this subgrid come to a total of $n+n-1$ squares. Thus, there are $(n-1)^2 + 2n - 1$ squares in an $n \times n$ grid.

(d) First, we peel off the leftmost column, and topmost row, removing exactly $2n-1$ squares. We then peel off the leftmost column and topmost row remaining, removing exactly $2(n-1)-1$ squares. We continue this process until we are left with a single square, which we also remove. This gives us a total of $(2n-1)+(2n-3)+\cdots+3+1 = \sum_{k=1}^{n} 2k-1$ squares in the $n \times n$ grid.