

**1. Balls and Bins.** Suppose you have  $m$  labeled balls  $a_1, \dots, a_m$  that you have thrown one by one, uniformly at random into  $n$  labeled bins  $b_1, \dots, b_n$ . For each of the probabilities below, select all answers that apply from the list of choices.

1. What is the probability that  $b_j$  contains  $a_i$ ? When  $a_i$  is being thrown, what is the probability that it lands in  $b_j$ ? How many choices are there?
2. What is the probability that  $b_j$  is empty? For each ball, what is the probability that it does not end up in  $b_j$ ? Are these events independent?
3. What is the probability that  $b_j$  contains all of the balls? For each ball, what is the probability that it ends up in  $b_j$ ? Are these events independent?
4. What is the probability that  $b_j$  contains exactly  $k$  balls? For each ball, what is the probability that it ends up in  $b_j$ ? Are these events independent? Does order matter?
5. What is the probability that  $b_j$  contains at most  $k$  balls? What is the probability that  $b_j$  contains exactly  $i$  balls?

**2. Processes, servers and overloading.** I have  $M$  processes (jobs) and  $N$  servers that I can assign the jobs to. Any job may be assigned to any server. Suppose I assign each job to a randomly chosen server, with all servers being equally likely. We say that a server is overloaded if it is assigned greater than or equal to  $K$  jobs, where  $K \leq M$ . What is the probability that the first server is overloaded?