

### 1. Homework Polling

Suppose Prof. Sahai wants to poll the CS 70 students about whether the homeworks have been too hard recently. Suppose everyone is comfortable enough to answer honestly, either no (0) or yes (1). Let the true fraction of students who think the homework is too hard be  $p$ . Let the response of the  $i^{\text{th}}$  polled student be  $X_i$ . Prof. Sahai would like to poll  $n$  students, with  $n$  large enough that the probability of estimating  $p$  to within  $\pm 2\%$  is at least 90%.

- a) Let  $M_n = \frac{1}{n} \sum_{i=1}^n X_i$  be the average of the  $n$  responses. What is the expectation of  $M_n$ ?  
What is the event that we are interested in whose probability we would like to be at least 90%? Draw a picture of the distribution of  $M_n - p$  and mark the region that corresponds to the event of interest.
  
- b) Now use Chebyshev's inequality to find a safe  $n$  regardless of what  $p$  is.
  
- c) What if instead of wanting an accuracy of  $\pm 2\%$  we wanted a relative error of 2%. This means that if the true value was  $p$ , we want the answer we get to be within  $[0.98p, 1.02p]$ . Can we pull that off using Chebyshev's inequality? Do you think we could pull that off with a single universal choice of  $n$  that does not depend on (the unknown)  $p$ ?

### 2. Will I Get My Package?

Sneaky delivery guy of some company is out delivering  $n$  packages to  $n$  customers. Not only does he hand a random package to each customer, he tends to open a package before delivering with probability  $\frac{1}{2}$ . Let  $X$  be the number of customers who receive their own packages unopened. Compute the expectation  $E(X)$  and the variance  $\text{Var}(X)$ .