EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai

Discussion 1W-S

1. Is it a proposition?

- $2+2=4$
- $x+2=4$
- Arnold Schwarzenegger is a handsome man.
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2. Quantifiers and Negation

a) Let $\mathbb{X} = \{\text{photos}\}$ and $\mathbb{Y} = \{\text{humans}\}$, which one of the following is equivalent to "All photos are taken by some human"?

$(\forall x \in \mathbb{X})(\forall y \in \mathbb{Y})(x \text{ is taken by } y)$

$(\forall x \in \mathbb{X})(\exists y \in \mathbb{Y})(x \text{ is taken by } y)$

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b) Let $\mathbb{Z}$ denote the set of all integers, and let $P(x)$ denote the proposition formula $x \geq 0$, which ones of the following are equivalent to "For every pair of integers, at least one of them is negative"?

$(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})(\neg P(x) \vee \neg P(y))$

$(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})\neg(P(x) \vee P(y))$

$\neg((\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(P(x) \wedge P(y))$

$(\forall x \in \mathbb{Z})\neg((\exists y \in \mathbb{Z})(P(x) \wedge P(y))$

3. Truth Table

Make the truth table for the boolean function

$$Y = (A \implies \neg B) \wedge (C \implies B).$$

Hint: Note that $P \implies Q$ is logically equivalent to $\neg P \vee Q$. Try converting $(A \implies \neg B)$ and $(C \implies B)$ to their equivalent disjuction forms first.

4. Direct Proofs

a) We call integer n an even number if and only if there exists an integer k, such that $n = 2k$. Prove that the negative of any even integer n is even.

b) Prove that the sum of any three consecutive integers is divisible by three.

5. Proof by Contraposition

Let x and y be two positive integers. Prove that if $x \times y < 36$ then $x < 6$ or $y < 6$.

6. Proof by Contradiction

a) The negative of any irrational number is irrational.

b) Prove that there are no inhabitants in town, given the following information:

1. No two inhabitants have the same number of hairs on their head.
2. No inhabitant is bald.
3. There are more inhabitants in town than hairs on any individual inhabitant's head.

c) Prove that if an implication is true, then its contrapositive is true.