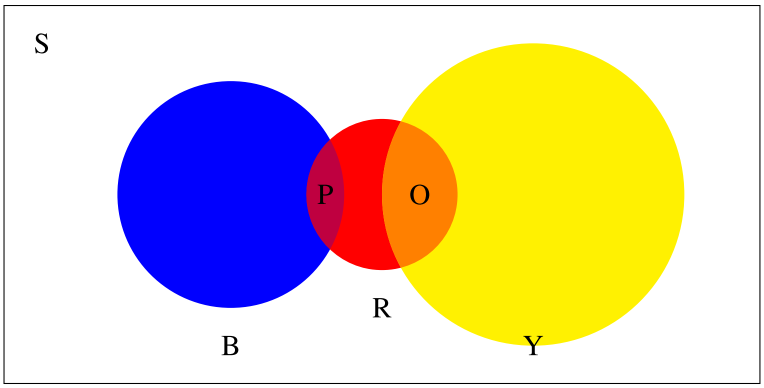
EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai **Discussion 8W-S**

1. Probability Space

Consider the drawing of the probability space S below. Here, the blue/purple region is the set of events B, the red/purple/orange region is the set of events R, and the yellow/orange region is the set of events Y. The set of events P is the set of events in both B and R, and is represented by the purple region. The set of events O is the set of events in both R and Y, and is represented by the orange region.



Assume that we are sampling from S uniformly at random.

(a) What is $\mathbb{P}[R]$, the probability that an element from S is in R?

(b) What is $\mathbb{P}[R|Y]$, the probability that an element from S is in R given that it is also in Y?

(c) What is $\mathbb{P}[R|O]$, the probability that an element from S is in R given that it is in O?

(d) What is $\mathbb{P}[P|B]$, the probability that an element from S is in P given it is in B?

(e) What is $\mathbb{P}[B \cup R \cup Y]$, the probability that an element of S is in B or R or Y?

(f) What is $\mathbb{P}[O|R \cup Y]$, the probability that an element of S is in O given that it is also in R or Y?

2. Random Treats

Suppose I have a bag of candy containing 10 chocolate bars, 5 lollipops and 5 toffees.

(a) If I randomly select a piece of candy to eat, what is the probability that it will be a chocolate bar?

(b) Suppose that I am trying to randomly select a candy for a friend who does not like chocolate, so that every time I choose a chocolate I return it to the bag, and I stop when I draw a candy that is not chocolate. What is the probability that I choose a toffee?

(c) Say that I have eaten one chocolate, one toffee, and one lollipop. True or false: now that I have eaten one of each candy, my probability of choosing a chocolate has decreased.

3. Bayesian Inference

In this problem, we will work through an example of Bayesian Inference.

Suppose you would like to decide whether to go to on a picnic tomorrow. You have some data about the weather in the area. You know the following:

- The probability of rain, $\mathbb{P}[R] = 0.2$
- The probability you see clouds the day before it rains, $\mathbb{P}[C|R] = 0.75$
- The probability you see clouds the day before a clear day, $\mathbb{P}[C|\overline{R}] = 0.1$

You notice heavy clouds in the sky. If you could calculate the probability that it will rain tomorrow conditioned on the clouds in the sky tonight, then you can make a more informed decision about tomorrow's plans.

(a) What is $\mathbb{P}[C \cap R]$?

(b) What is $\mathbb{P}[C \cap \overline{R}]$?

(c) What is $\mathbb{P}[R|C]$?

(d) Your prior was $\mathbb{P}[R]$, the probability that it rains tomorrow. Assuming you do not want to get rained on, will you be more or less likely to go on a picnic tomorrow considering your posterior probability $\mathbb{P}[R|C]$?