

1. Pokemon Counting!

- (a) I have caught 30 different Pokemon¹ so far. In how many ways can I choose a team of 6, such that the order of my team matters?
- (b) For this part and the next two, you can assume we no longer care about the order of the Pokemon. In how many ways can I choose a team of 6 under this assumption?
- (c) Among my 30 caught Pokemon, only 4 can learn the move “Fly”. In how many ways can I choose a team of 6, such that there is **exactly** one Pokemon who knows the move “Fly”?
- (d) In how many ways can I choose a team of 6, such that there is **at least** one Pokemon who knows the move “Fly”?
- (e) You were victorious against the Elite Four, and Professor Oak generously invited all six members of your team to the Hall of Fame. Suppose he wants to sit your team in a circular table for dinner (which consists of only Oran Berries), in how many ways can he do so?
- (f) Suppose Charizard and Pikachu, two members of your team, want to sit next to each other. How would your answer to the above question change?

¹Pokemon is an abbreviation of Pocket Monsters. Hence, the plural of Pokemon is still Pokemon...

- (g) Suppose Meowth and Pikachu, two members of your team, don't want to sit next to each other. In how many ways can Professor Oak arrange the seatings?

2. Pokemon Anagrams

An anagram of a word is any re-ordering of the letters of the word, in any order. It does not have to be an English word or an actual Pokemon name.

- (a) How many different anagrams of PIKACHU are there?

- (b) How many different anagrams of KADABRA are there?

- (c) How many different anagrams of RATTATA are there?

3. Pokemon Levels

For this question, assume that a Pokemon's *level* is a **nonnegative integer**, unless stated otherwise.²

- (a) Suppose you need to train your team (technical word: "level grinding") until the total levels of all six Pokemon is exactly 20 to qualify for your first Gym battle. Let x_i denotes the level of the i^{th} Pokemon. How many nonnegative integer solutions are there to this equation?

$$x_0 + x_1 + x_2 + x_3 + x_4 + x_5 = 20$$

²This is actually not true in the game. See the last part.

(b) In general, how many solutions does

$$x_0 + x_1 + \dots + x_k = n$$

have, if all x_i must be non-negative integers?

(c) Suppose I only need to “grind” two of my Pokemon for a Double battle challenge. How many solutions does

$$x_0 + x_1 = n$$

have, if every x_i must be *strictly positive* integers?

(d) What if I need to train all of my n Pokemon, as seen in part (b)? How many solutions does

$$x_0 + x_1 + \dots + x_k = n$$

have, if every x_i must be *strictly positive* integers?

(e) Most of the constraints above are actually incorrect in the game – a Pokemon cannot have level 0, and most Pokemon caught in the wild or bred from eggs actually start at level 5. Using your knowledge from the above parts, solve for the number of solutions of

$$x_0 + x_1 + x_2 + x_3 + x_4 + x_5 = 100$$

where $x_i \geq 5$.