1. Cookies Again

GSIs still have plenty of cookies left from last section, so we are giving out cookies again! To make it more interesting, we give exactly 10 cookies to a student with probability $1/2$, otherwise we give either 1, 2, or 3 cookies randomly with equal probability. What is the expected number of cookies you'd get? What is its variance?

2. Pokemon Again

Nintendo comes out with a set of reprinted original Pokemon cards, one for each of the original 150 Pokemon. Naturally nothing will do, but that you collect them all. Suppose that these cards are sold as individually wrapped random cards and all cards are equally common. You can't see what a card is until after you buy it.

(a) Approximately how many cards would you expect to need to buy before you have all 150?

(b) Now suppose you have a nice older brother who still has 25 of his original cards (all unique). If he gives those cards to you, what is the expected number of cards you need to buy now?

(c) Your older brother isn't that nice; he won't give you his cards for free but he will sell them. If new cards go for \$2.50 a piece, how much would you be willing to pay your brother for his cards? Is the per-card price greater or less than \$2.50? Why?

3. EECS 0070

James Bond is imprisoned in a cell from which there are three possible ways to escape: an air-conditioning duct, a sewer pipe and the door (which is unlocked). The air-conditioning duct leads him on a two-hour trip whereupon he falls through a trap door onto his head, much to the amusement of his captors. The sewer pipe is similar but takes five hours to traverse. Each fall produces amnesia and he is returned to the cell immediately after each fall. Assume that he always immediately chooses one of the three exits from the cell with probability $\frac{1}{3}$. On average, how long does it take before he opens the unlocked door and escapes?

4. Would You Bet It? v2

Suppose EECS70 offers yet another game: you bet any amount of homework points, then we flip a coin and if it comes up heads you win that amount, and if it comes up tails you lose that amount.

Suppose you follow this strategy: you start with a bet of 1 homework point, and if you lose you increase your bet to 2 homework points, and again if you lose you double your bet to 4 homework points, and so on. As soon as you win, you take your winnings and you go out (i.e. you bet 0 homework points for the next rounds). So in round n you bet $2^n 1$ homework points if you have lost all the previous rounds, and you bet 0 homework points if you have won any of the previous rounds.

(a) What is your net winnings (i.e. subtract your losses from your wins) if you win after the n-th coin flip.

(b) What is the expected value of your winnings on round i?

(c) What is you expected net winnings after round n?

5. Prove it

Assume the random variable X takes on integer values from $1, 2, \ldots n-1, n$. Prove that

$$E[X] = \sum_{i=1}^{n} P(X \geq i).$$