

### 1. DeMorgan's Law

Use truth tables to show that  $\neg(A \vee B) \equiv \neg A \wedge \neg B$  and  $\neg(A \wedge B) \equiv \neg A \vee \neg B$ . These two equivalences are known as DeMorgan's Law.

### 2. Notation Practice

Write the following statements using the notation covered in class. (Use  $\mathbb{N}$  to denote the set of natural numbers and  $\mathbb{Z}$  to denote the set of integers. Also write  $P(n)$  for the statement “ $n$  is odd”.)

1. For all natural numbers  $n$ ,  $2n$  is even.
2. For all natural numbers  $n$ ,  $n$  is odd if  $n^2$  is odd.
3. There are no integer solutions to the equation  $x^2 - y^2 = 10$ .

### 3. Infinite Primes

Prove by contradiction that there are an infinite number of primes.

#### 4. Prime Form

Prove that every prime number  $m > 3$  is either of the form  $6k + 1$  or  $6k - 1$  for some integer  $k$ .

#### 5. Numbers of Friends

If there are  $n > 2$  people at a party, then at least 2 of them have the same number of friends at the party. Prove this by contradiction.

#### 6. The Triangle Inequality

You may remember from a previous math class the *triangle inequality*, which states that for real numbers  $x_1$  and  $x_2$ ,

$$|x_1 + x_2| \leq |x_1| + |x_2|.$$

Generalize the triangle inequality using mathematical induction to prove that

$$|x_1 + x_2 + \dots + x_n| \leq |x_1| + |x_2| + \dots + |x_n|.$$