1. Chinese Remainder Theorem

For each system of modular equations, indicate what mod the solution is in, or state that it has no solution.

1.

$$x \equiv 1 \mod 3$$
$$x \equiv 3 \mod 4$$
$$x \equiv 2 \mod 5$$

2.

$$x \equiv 2 \mod 8$$
$$x \equiv 4 \mod 12$$
$$x \equiv 16 \mod 22$$

3.

$$x \equiv 2 \mod 6$$
$$x \equiv 3 \mod 9$$
$$x \equiv 9 \mod 15$$

4.

$$x \equiv 2 \mod 4$$
$$x \equiv 3 \mod 9$$
$$x \equiv 12 \mod 30$$

2. Fermat's Little Theorem

One form of Fermat's Little Theorem states that if p is a prime and if a is an integer, then

$$a^p \equiv a \mod p$$

which is equivalent to

$$a^{p-1} \equiv 1 \mod p$$

1. Now we will try to use Fermat's Little Theorem to compute $128^{129} \mod 17$. In this case,
   $a =$
   $p =$
   Now, simplify $128^{129} \mod 17$.

2. Similary, calculate $x = 2^{345} \mod 11$ efficiently.