EECS 70 Discrete Mathematics and Probability Theory
Spring 2014 Anant Sahai

# Final Review Sol

**1. True/False**

Circle the right answer. No justification is needed.

(T) **F** $\forall x\,(P(x) \Rightarrow \exists y\, Q(y)) \equiv \neg\exists x\,(P(x) \wedge \forall y \neg Q(y))$

**T** (F) $\exists i, \forall j, P(i,j) \implies \forall i, \exists j, \neg P(i,j)$.

**T** (F) Let $P(x) =$"$x$ is prime" and $Q(x) =$"$x$ is even". It is true that: $\neg\exists x\,(P(x) \wedge Q(x) \Rightarrow x = 2)$.

(T) **F** For $P, Q$ as above the following is true: $\forall x\,(P(x) \wedge Q(x) \Rightarrow x = 2)$

**T** (F) 6 has a multiplicative inverse modulo 15.

(T) **F** The efficient implementation of RSA hinges upon our ability to efficiently check whether a number is prime or not.

(T) **F** Toby and his 4 friends go to a horror movie and sit together in five consecutive seats. Toby will not sit in the middle seat. The number of ways the 5 friends can be arranged in the 5 seats is 96.

(T) **F** For any two disjoint events $A, B$, with $\Pr[B] \neq 0$, it holds that $\Pr[A|B] = 0$.

(T) **F** For any set of $n$ i.i.d. random variables it holds that $E[X_1 \cdot X_2 \ldots \cdot X_n] = E[X_1]^n$.

(T) **F** The union bound $\Pr[\bigcup_{i=1}^n A_i] \leq \sum_{i=1}^n \Pr[A_i]$ holds for all events $A_i$, disjoint or not.

**T** (F) The number of cereal boxes we have to buy before collecting all $n$ different baseball cards follows the Geometric distribution with parameter $1/n$.

(T) **F** If you have a set of 11 points, where 7 of them agree with a degree 4 polynomial $p_1$, and 9 of them agree with a degree 4 polynomial $p_2$, then $p_1$ and $p_2$ must be the same polynomial.

**T** (F) The set of reals is countable.

(T) **F** The set of all subsets of size 10 of the integers is countable.

**T** (F) The set of all subsets of the integers is countable.

**T** (F) The set of all finite subsets of the natural numbers is uncountable.

(T) **F** For any events $A$ and $B$, if $\mathbf{P}[A] \neq 0$, $\mathbf{P}[B] \neq 0$, and $A$ and $B$ disjoint, then $A$ and $B$ are dependent.

**T** (F) For any two events $A$ and $B$, if $\mathbf{P}[A] \neq 0$, $\mathbf{P}[B] \neq 0$, and $\mathbf{P}[A|B] = 1$, then $\mathbf{P}[B|A] = 1$.

**T** (F) For any two events, if $\mathbf{P}[B] \neq 0$ and $\mathbf{P}[\overline{B}] \neq 0$, then $P[A|B] + P[A|\overline{B}] = 1$.

**T** (F) For any three events, $A$, $B$, $C$, if $\mathbf{P}[A] \neq 0$, $\mathbf{P}[B] \neq 0$, and $A$ and $B$ are independent, then $A$ and $B$ are conditionally independent on $C$.