CS 70 Discrete Mathematics and Probability Theory
Fall 2015 Jean Walrand Discussion 12A

1. **(Sanity Check!)** Prove or give a counterexample: for any random variables X and Y, $\text{Var}[\text{X}+\text{Y}] = \text{Var}[\text{X}]+\text{Var}[\text{Y}]$.

2. **(Bernoulli and Binomial Distribution)** A random variable X is called a Bernoulli random variable with parameter p if $X = 1$ with probability p and $X = 0$ with probability $1-p$.

   (a) Calculate $\text{E}[X]$ and $\text{Var}[X]$.

   (b) A Binomial random variable with parameters n and p is defined to be the sum of n independent, identically distributed Bernoulli random variables with parameter p. If Z is a Binomial random variable with parameters n and p, what are $\text{E}[Z]$ and $\text{Var}[Z]$?

3. **(Telebears)** Lydia has just started her Telebears appointment. She needs to register for a marine science class and CS70. There are no waitlists, and she can attempt to enroll once per day in either class or both. The Telebears system is strange and picky, so the probability of enrolling in the marine science class is p_1 and the probability of enrolling in CS70 is p_2. The probabilities are independent. Let M be the number of attempts it takes to enroll in the marine science class, and C be the number of attempts it takes to enroll in CS70.

   (a) What distribution do M and C follow? Are M and C independent?

   (b) For an integer $k \geq 1$, what is $\Pr[C \geq k]$?

   (c) What is the expected number of classes she will be enrolled in if she must enroll with 14 days (inclusive)?

(d) For an integer $k \geq 1$, what is the probability that she is enrolled in both classes before attempt k?

4. **(Toujours les poissons)** Use the Poisson distribution to answer these questions.

   (a) Suppose that on average, 20 people ride your roller coaster per day. What is the probability that exactly 7 people ride it tomorrow?

   (b) Suppose that on average, you go to Six Flags twice a year. What is the probability that you will go at most once in 2015?

   (c) Suppose that on average, there are 5.7 accidents per day on California roller coasters. (I hope this is not true.) What is the probability there will be *at least* 3 accidents throughout the *next two days* on California roller coasters?

5. **(Will I Get My Package?)** A sneaky delivery guy of some company is out delivering n packages to n customers. Not only does he hand a random package to each customer, he tends to open a package before delivering with probability $\frac{1}{2}$ (independently of the choice of the package). Let X be the number of customers who receive their own packages unopened.

   (a) Compute the expectation $\mathrm{E}(X)$.

   (b) What is the probability that customers i and j both receive their own packages unopened?

   (c) Compute the variance $\mathrm{Var}[\mathrm{X}]$.