EECS 70 Discrete Mathematics and Probability Theory
Fall 2015 Satish Rao

Discussion 2B

1. (Contraposition) Prove that if $a+b<c+d$, then $a<c$ or $b<d$.

2. (Problem formulation) Write the following statements using the notation covered in class. Use $\mathbb{N}$ to denote the set of natural numbers and $\mathbb{Z}$ to denote the set of integers. Also write $P(n)$ for the statement "n is odd".

a) For all natural numbers n, $2n$ is even.
b) For all natural numbers n, n is odd if n^2 is odd.
c) There are no integer solutions to the equation $x^2-y^2=10$.

3. (Induction) Prove that, for any positive integer n, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

4. (Problem solving) Prove that the length of the hypotenuse of a non-degenerate right triangle is strictly less than the sum of the two remaining sides.

1. Write down the definition of a right triangle and the claim to be proven in mathematical notation.
2. Prove the statement by contradiction.
3. Prove the statement directly.

5. (Problem solving) You may have seen the series $1+\frac{1}{2}+\frac{1}{3}+\ldots$ in calculus. This is known as a *harmonic series*, and it diverges, i.e. the sum approaches infinity. We are going to prove this fact using induction.

Let $H_j = \sum_{k=1}^{j} \frac{1}{k}$. Use mathematical induction to show that, for all integers $n \geq 0$, $H_{2^n} \geq 1+\frac{n}{2}$, thus showing that H_j must grow unboundedly as $j \to \infty$.