Today

Review for Midterm.

First there was logic...

A statement is a true or false.

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Statements?

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Statements?

$3 = 4 - 1$?

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Statements?

$3 = 4 - 1$? Statement!

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$3 = 4 - 1$? Statement!

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$n = 3$?

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$n = 3$? Not a statement...but a predicate.

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Predicate: Statement with free variable(s).

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Example: $x = 3$

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$n > 3$?

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Example: $x = 3$ Given a value for x, becomes a statement.

Predicate?

$n > 3$? Predicate: $P(n)$!

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Example: $x = 3$ Given a value for x, becomes a statement.

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$x = y$?

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For every x, $P(x)$ is true.

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Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$.

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You got this!

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Propositional Expressions and Logical Equivalence

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

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$A \wedge B$, $A \vee B$, $\neg A$.

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

Proofs: truth table or manipulation of known formulas.

Connecting Statements

$A \wedge B$, $A \vee B$, $\neg A$.

You got this!

Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

Proofs: truth table or manipulation of known formulas.

$(\forall x)(P(x) \wedge Q(x)) \equiv (\forall x)P(x) \wedge (\forall x)Q(x)$

..and then proofs...

Direct: $P \implies Q$

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Example: a is even $\implies a^2$ is even.

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Approach: What is even?

..and then proofs...

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$$a^2 = 2(2k^2)$$

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$\neg P \implies$ **false**

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$\neg P \implies R \wedge \neg R$

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Useful for prove something does not exist:

..and then proofs...

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Example: rational representation of $\sqrt{2}$

..and then proofs...

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Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$ does not exist.

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Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$ does not exist.

Example: finite set of primes

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Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$ does not exist.

Example: finite set of primes does not exist. Example: rogue couple does not exist.

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...jumping forward..

Contradiction in induction:

...jumping forward..

Contradiction in induction:

contradict place where induction step doesn't hold.

...jumping forward..

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Well Ordering Principle.

...jumping forward..

Contradiction in induction:
contradict place where induction step doesn't hold.

Well Ordering Principle.
Stable Marriage:

...jumping forward..

Contradiction in induction:
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Well Ordering Principle.
Stable Marriage:
first day where women does not improve.

...jumping forward..

Contradiction in induction:
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Well Ordering Principle.
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first day where any man rejected by optimal women.

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Contradiction in induction:
contradict place where induction step doesn't hold.

Well Ordering Principle.
Stable Marriage:
first day where women does not improve.
first day where any man rejected by optimal women.
Do not exist.

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...and then induction...

$$P(0) \wedge ((\forall n)(P(n) \implies P(n+1) \equiv (\forall n \in N)\, P(n).$$

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Thm: For all $n \geq 1$, $8 | 3^{2n} - 1$.

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Induction on n.

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$3^{2n+2} - 1 = 9(3^{2n}) - 1$

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Thm: For all $n \geq 1$, $8|3^{2n} - 1$.

Induction on n.

Base: $8|3^2 - 1$.

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$(3^{2n} - 1 = 8d)$

Induction Step:

$3^{2n+2} - 1 = 9(3^{2n}) - 1$ (by induction hypothesis)

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$\square$

Stable Marriage: a study in definitions and WOP.

n-men, n-women.

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A m_j and w_k who like each other more than their partners

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Does stable pairing exist?

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Does stable pairing exist?

No, for roommates problem.

TMA.

Traditional Marriage Algorithm:

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Each Day:

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Every day, if man on string for woman, any future man on string is better.

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Not rogue couple!

Optimality/Pessimal

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Not necessarily first in list.

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Thm: woman pessimal.

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Thm: woman pessimal.

  Man optimal $\implies$ Woman pessimal.

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Thm: woman pessimal.

Man optimal $\implies$ Woman pessimal.
Woman optimal $\implies$ Man pessimal.

...Graphs...

$$G = (V, E)$$

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V - set of vertices.

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Thm: Sum of degrees is $2|E|$.

...Graphs...

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Thm: Sum of degrees is $2|E|$.
Edge is incident to 2 vertices.

...Graphs...

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$E \subseteq V \times V$ - set of edges.

Directed: ordered pair of vertices.

Adjacent, Incident, Degree.
In-degree, Out-degree.

Thm: Sum of degrees is $2|E|$.
Edge is incident to 2 vertices.
Degree of vertices is total incidences.

...Graphs...

$G = (V, E)$
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Connected Graph: one connected component.

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Thm: Every connected graph where every vertex has even degree has an Eulerian Tour; a tour which visits every edge exactly once.

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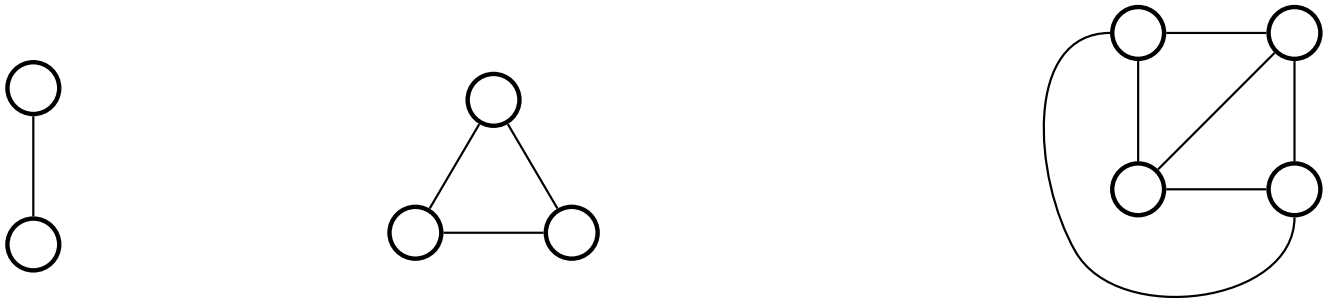

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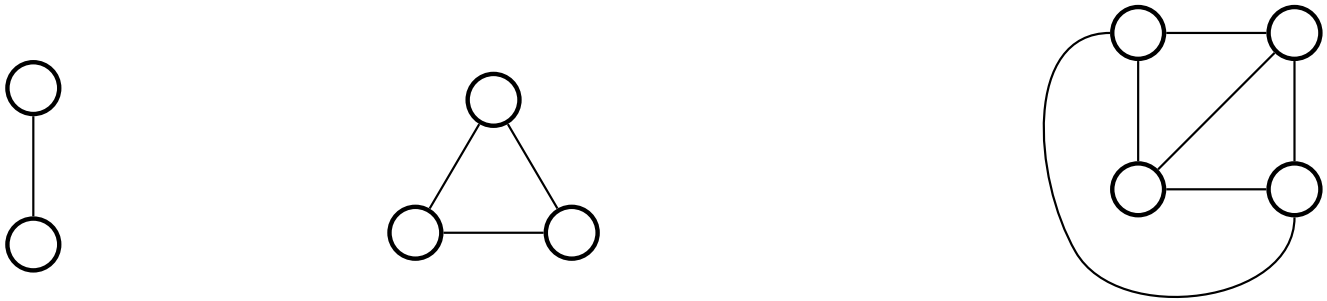

K_n, $|V| = n$

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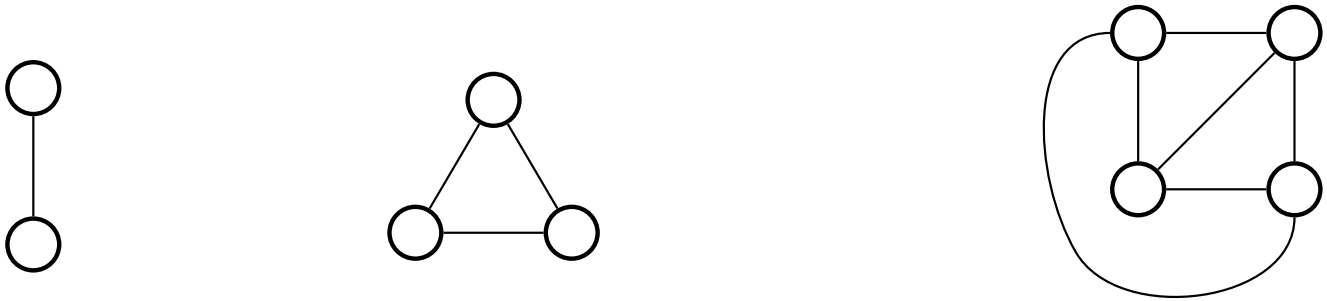

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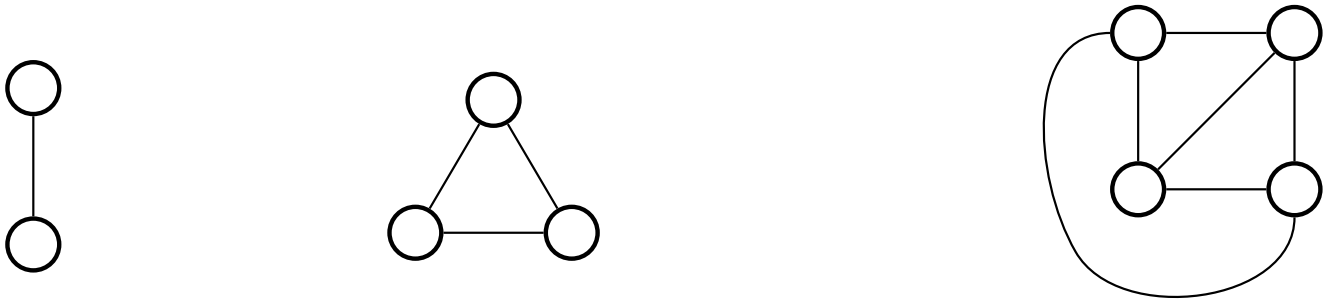

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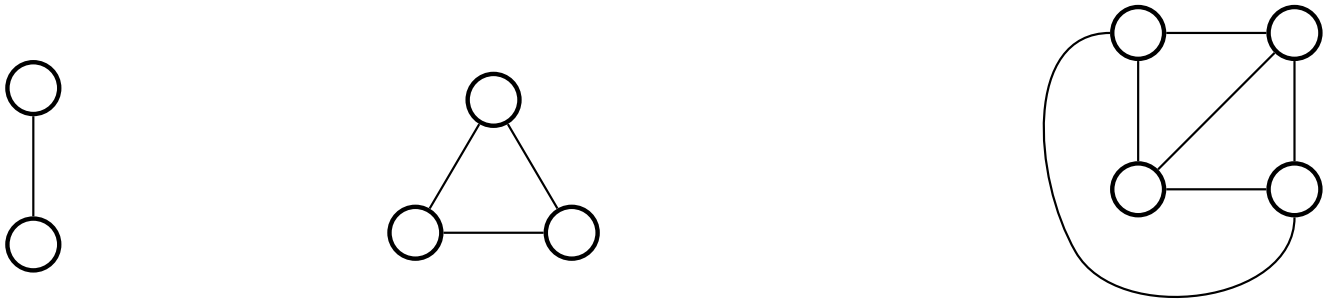

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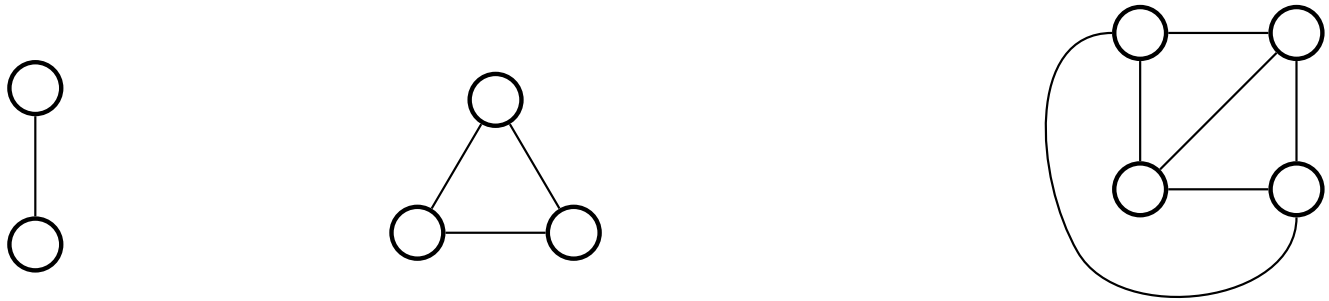
degree of vertex? $|V| - 1$.

Graph Types: Complete Graph.



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degree of vertex? $|V| - 1$.

Very connected.

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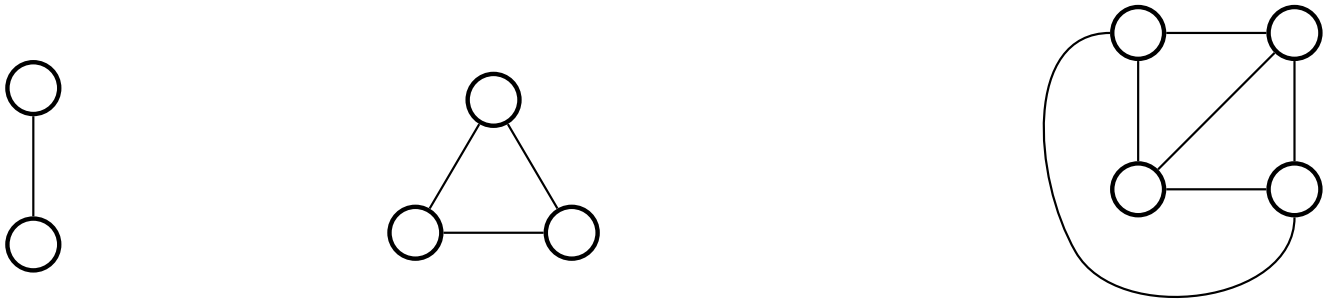
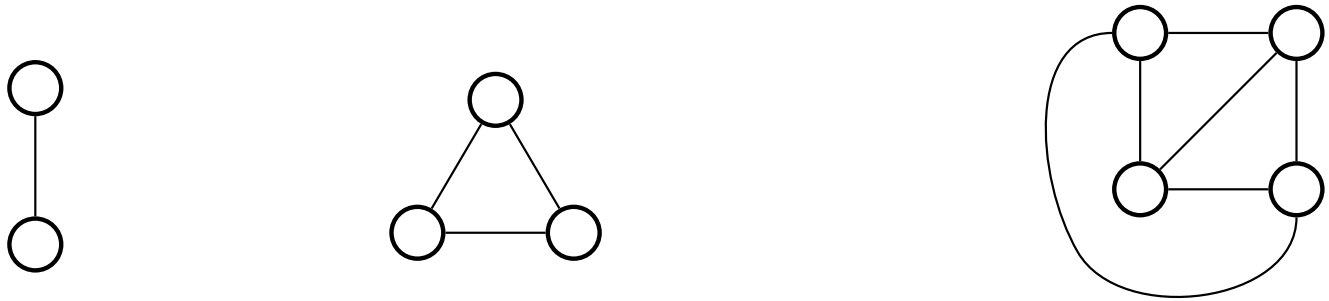


K_n, $|V| = n$

every edge present.
degree of vertex? $|V| - 1$.

Very connected.
Lots of edges:

Graph Types: Complete Graph.

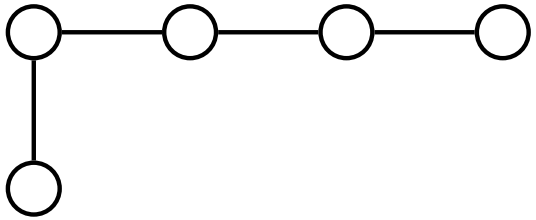


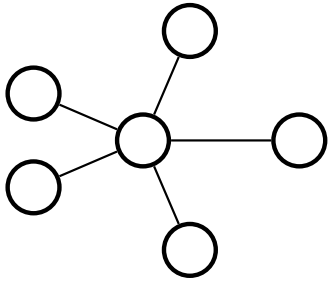
K_n, $|V| = n$

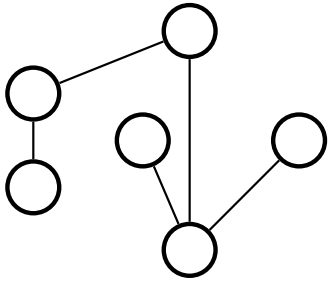
every edge present.
degree of vertex? $|V| - 1$.

Very connected.
Lots of edges: $n(n-1)/2$.

Trees.







Definitions:

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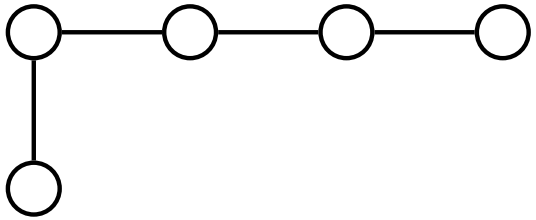
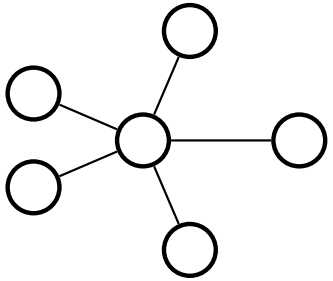
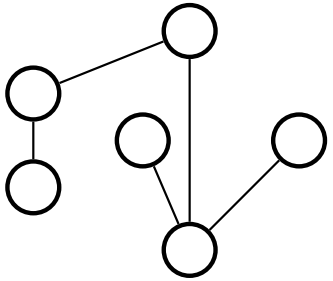


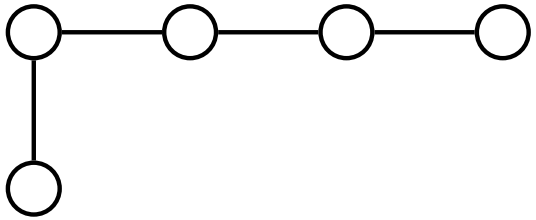


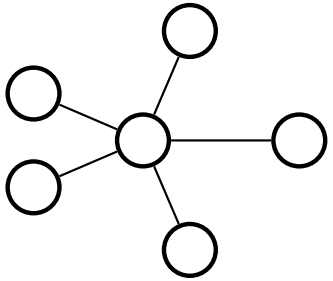


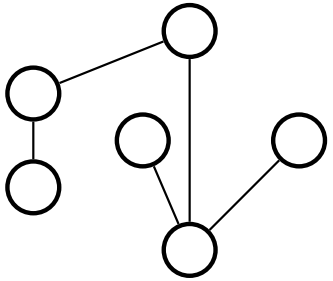
Definitions:

A connected graph without a cycle.

Trees.







Definitions:

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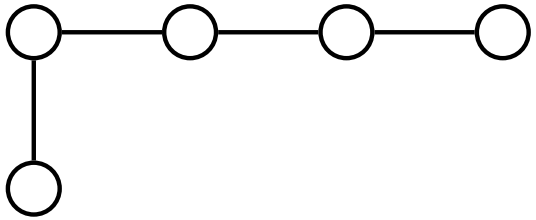
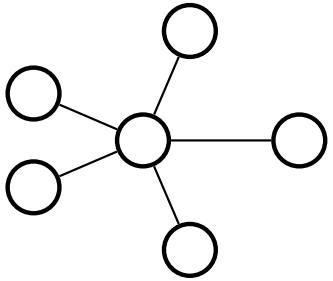
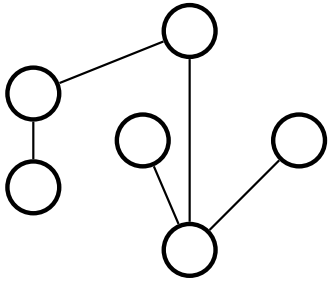
A connected graph with $|V| - 1$ edges.

Trees.







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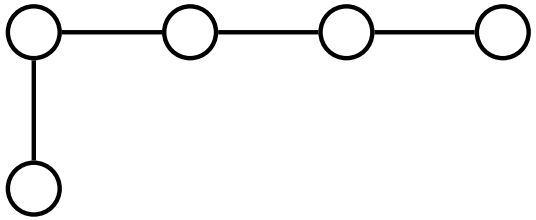
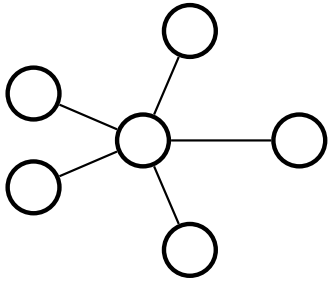
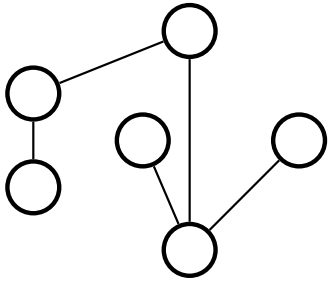
A connected graph where any edge removal disconnects it.

Trees.







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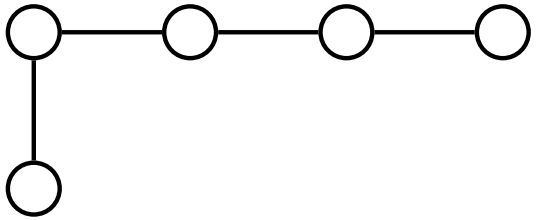

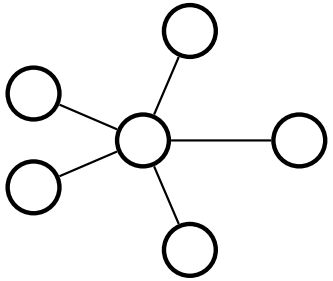

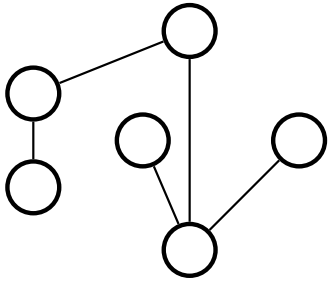

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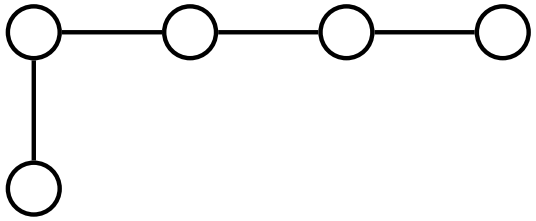
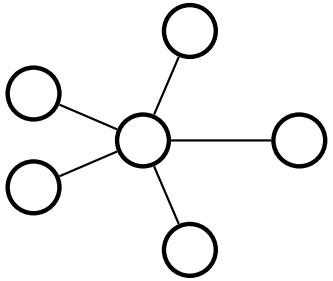
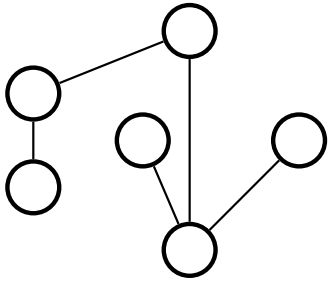

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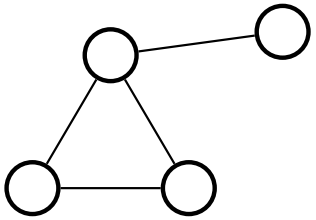
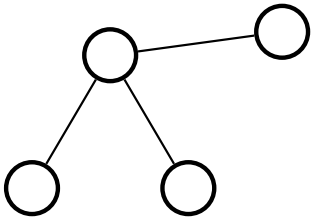
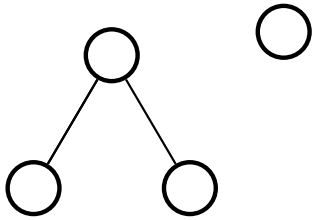

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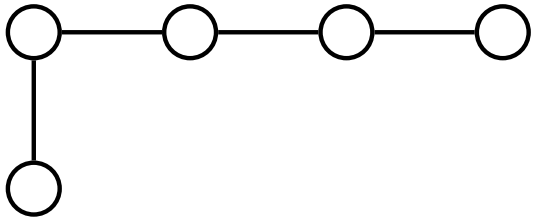
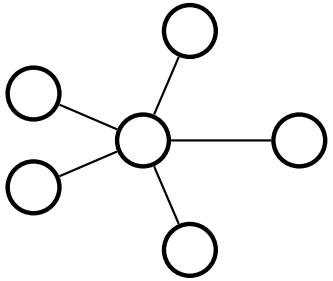
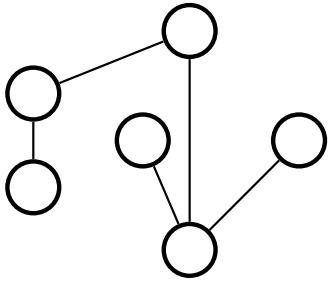

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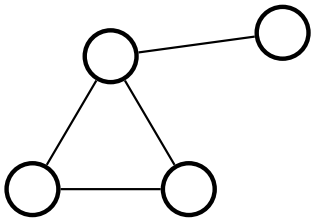
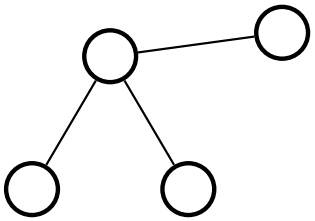
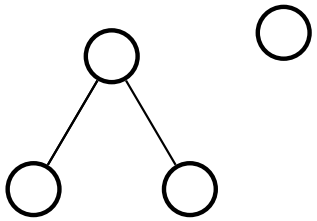
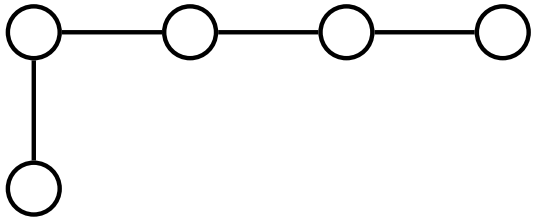
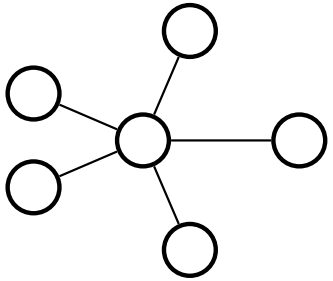
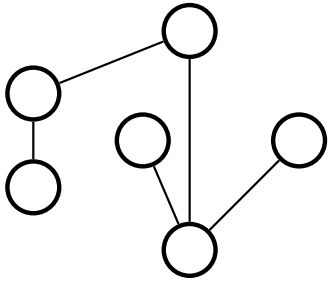




Minimally connected, minimum number of edges to connect.

Trees.

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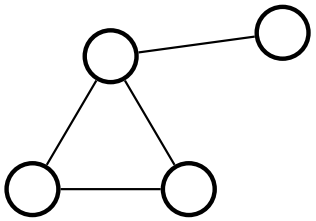
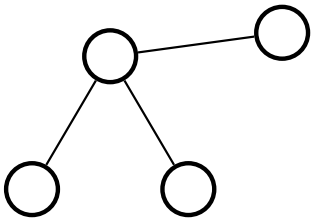
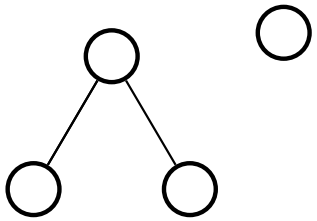
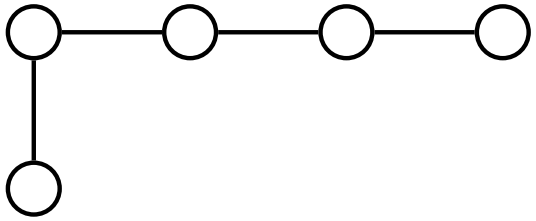
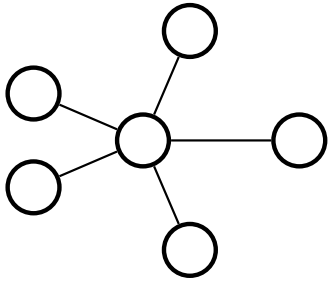
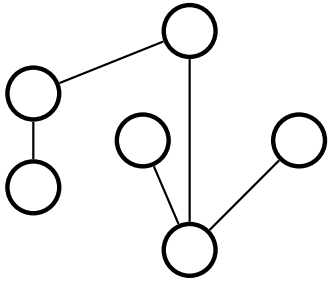




Minimally connected, minimum number of edges to connect.

Property:

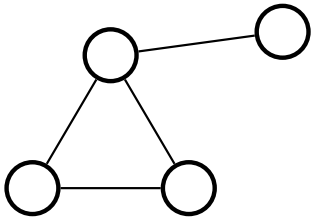
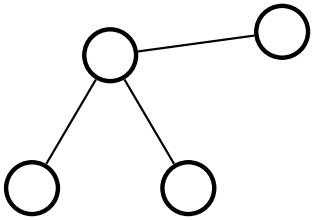
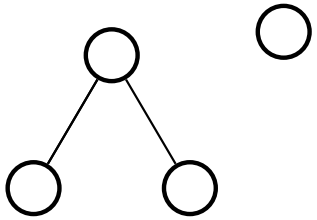
Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.
An acyclic graph where any edge addition creates a cycle.

To tree or not to tree!

Minimally connected, minimum number of edges to connect.

Property:
Can remove a single node and break into components of size at most $|V|/2$.

Hypercube

Hypercubes.

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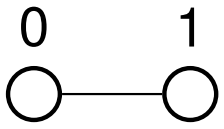

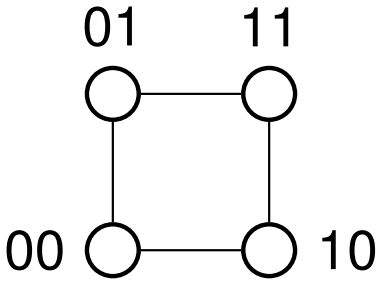

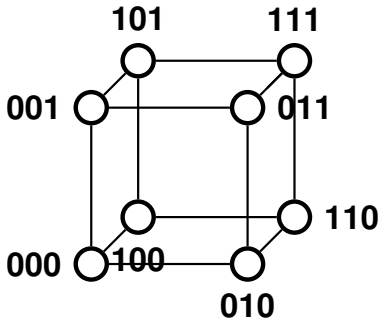

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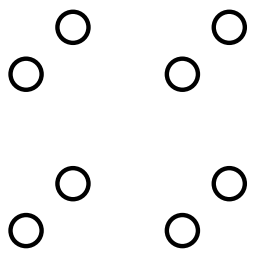

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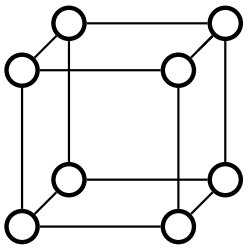

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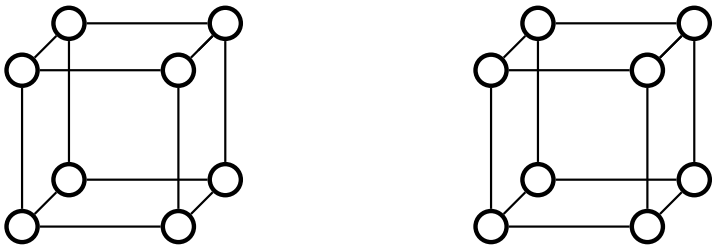

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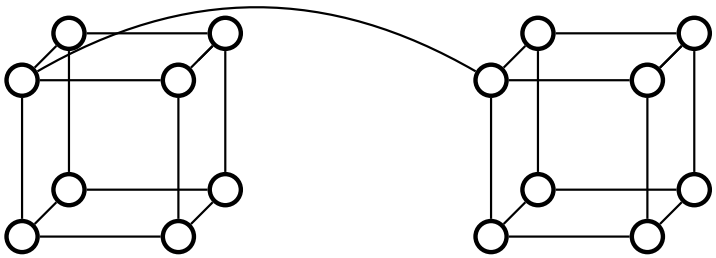

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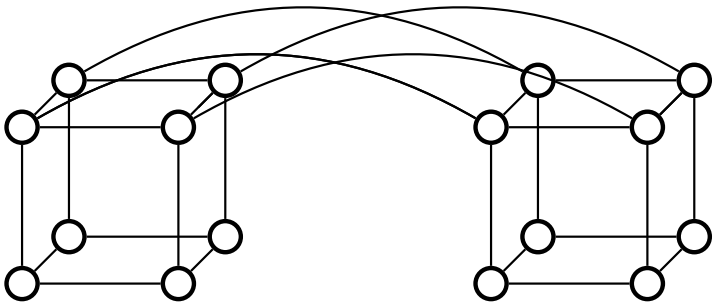

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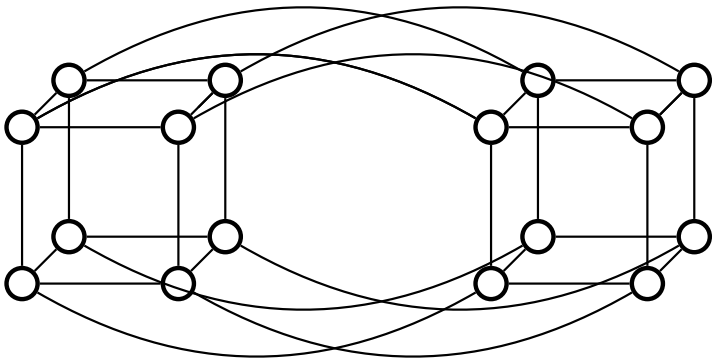

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egcd$(x, m) = (1, a, b)$

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Extended-gcd(x, y) returns (d, a, b)
$d = gcd(x, y)$ and $d = ax + by$

Multiplicative inverse of (x, m).
egcd$(x, m) = (1, a, b)$
a is inverse!

Modular Arithmetic Inverses and GCD

x has inverse modulo m if and only if $gcd(x, m) = 1$.

Group structures more generally.

Proof Idea:
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See piazza for another wonderful view from Zheng Zhu.

Midterm format

Time: 80 minutes.

Midterm format

Time: 80 minutes.

Many short answers.

Midterm format

Time: 80 minutes.

Many short answers.
  Get at ideas that we study.

Midterm format

Time: 80 minutes.

Many short answers.
  Get at ideas that we study.
   Know material well:

Midterm format

Time: 80 minutes.

Many short answers.
Get at ideas that we study.
Know material well: fast,

Midterm format

Time: 80 minutes.

Many short answers.
Get at ideas that we study.
Know material well: fast, correct.

Midterm format

Time: 80 minutes.

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Midterm format

Time: 80 minutes.

Many short answers.
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Midterm format

Time: 80 minutes.

Many short answers.
  Get at ideas that we study.
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Midterm format

Time: 80 minutes.

Many short answers.

Get at ideas that we study.

Know material well:	fast, correct.
Know material medium:	slower, less correct.
Know material not so well:	

Midterm format

Time: 80 minutes.

Many short answers.

Get at ideas that we study.

Know material well:	fast, correct.
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Know material not so well:	Uh oh.

Midterm format

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Some longer questions.

Midterm format

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Wrapup.

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Watch Piazza for Logistics!

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If you sent me email about Midterm conflicts

Wrapup.

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If you sent me email about Midterm conflicts
Other arrangements.

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Wrapup.

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Other issues....

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