Introduction

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Jean Walrand – Prof. of EECS – UCB
257 Cory Hall – walrand@berkeley.edu

I was born in Belgium(1) and came to Berkeley for my PhD. I have been teaching at UCB since 1982.

My wife and I live in Berkeley. We have two daughters (UC alumni – Go Bears!). We like to ski and play tennis (both poorly). We enjoy classical music and jazz.

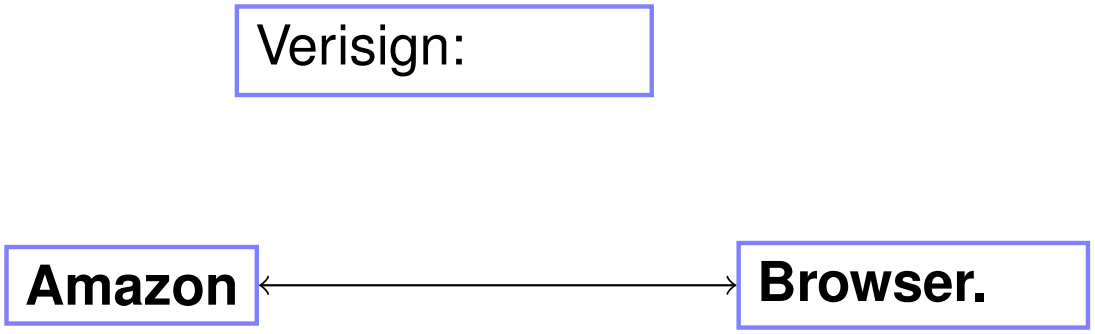
My research interests include stochastic systems, networks and game theory.

(1)

Lecture 13. Outline.

1. Signature Schemes
2. Polynomials
3. Secret Sharing
4. Polynomial Interpolation

Signatures using RSA.



Signatures using RSA.

Verisign:

Amazon ⟷ **Browser.**

Certificate Authority: Verisign, GoDaddy, DigiNotar,...

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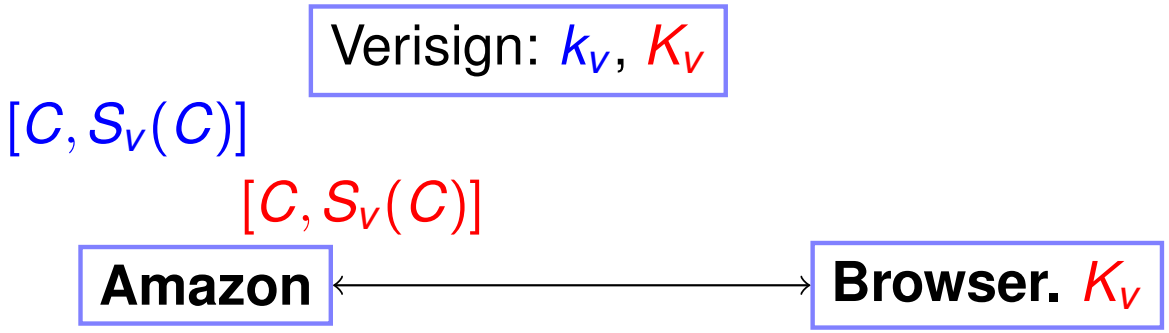

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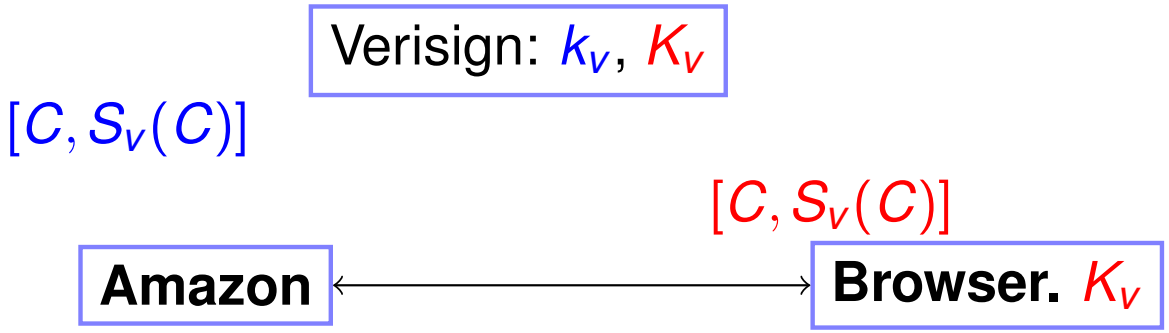

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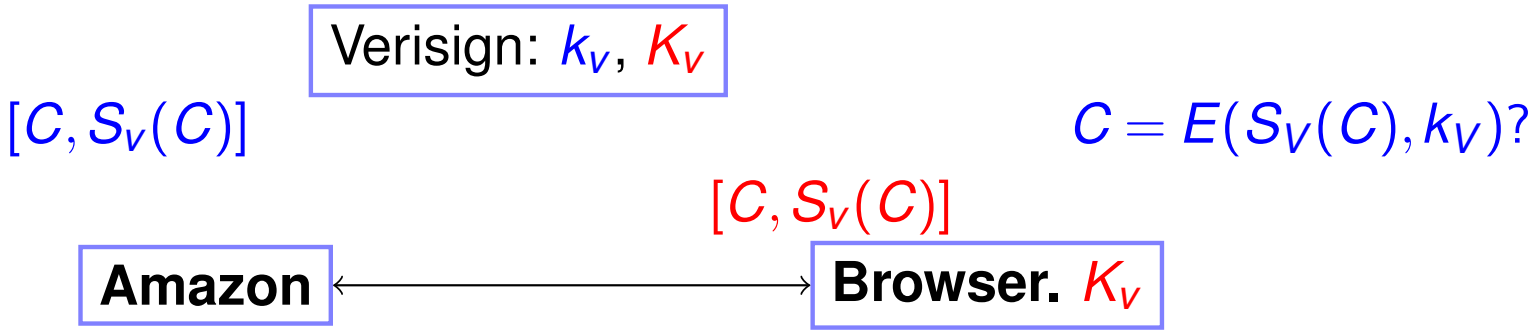

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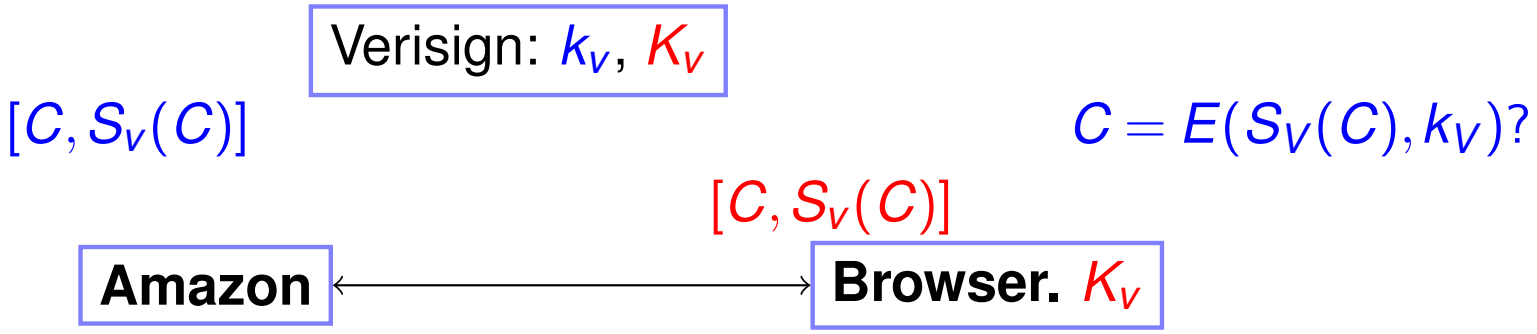

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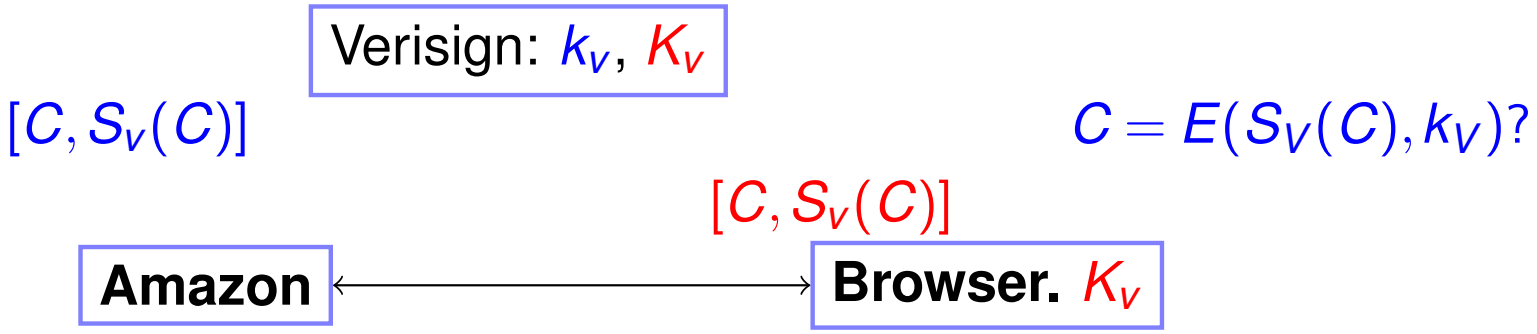

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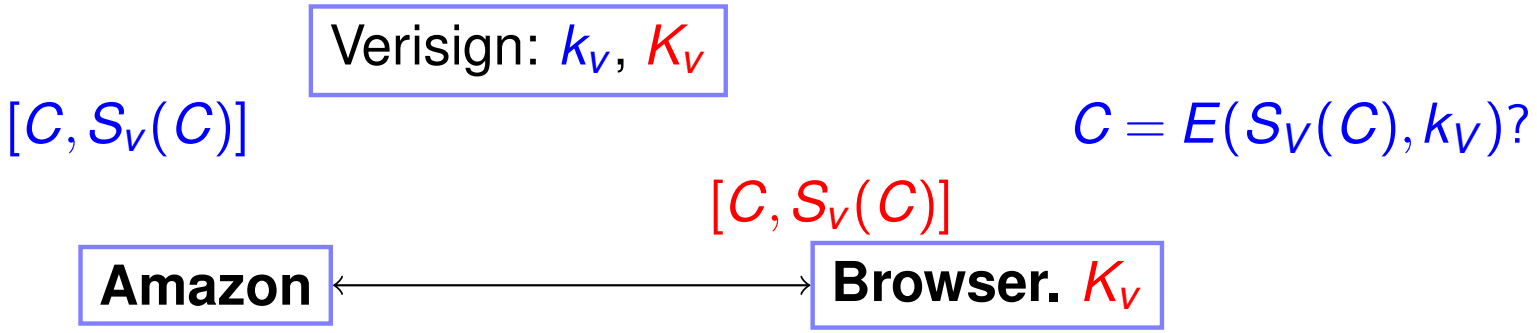

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Security: Eve can't forge unless she "breaks" RSA scheme.

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Polynomials

A **polynomial**

$$P(x) = a_d x^d + a_{d-1} x^{d-1} \cdots + a_0.$$

is specified by **coefficients** $a_d, \ldots a_0$.

[1] A field is a set of elements with addition and multiplication operations, with inverses. $GF(p) = (\{0, \ldots, p-1\}, + \pmod p, * \pmod p)$.

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Polynomials $P(x)$ **with arithmetic modulo** p: [1] $a_i \in \{0, \ldots, p-1\}$ and

$$P(x) = a_d x^d + a_{d-1} x^{d-1} \cdots + a_0 \pmod{p},$$

for $x \in \{0, \ldots, p-1\}$.

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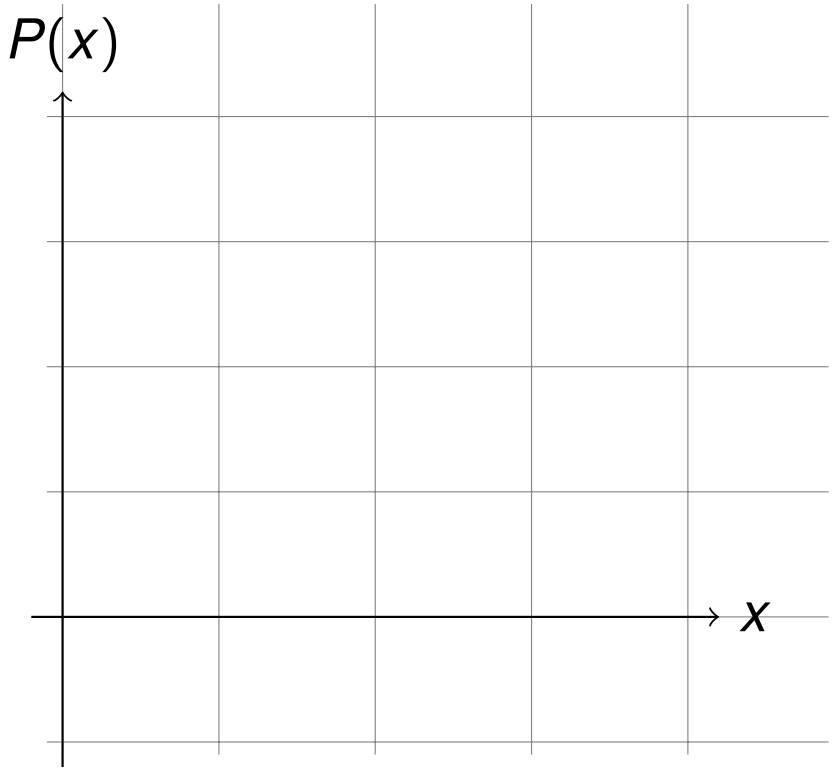

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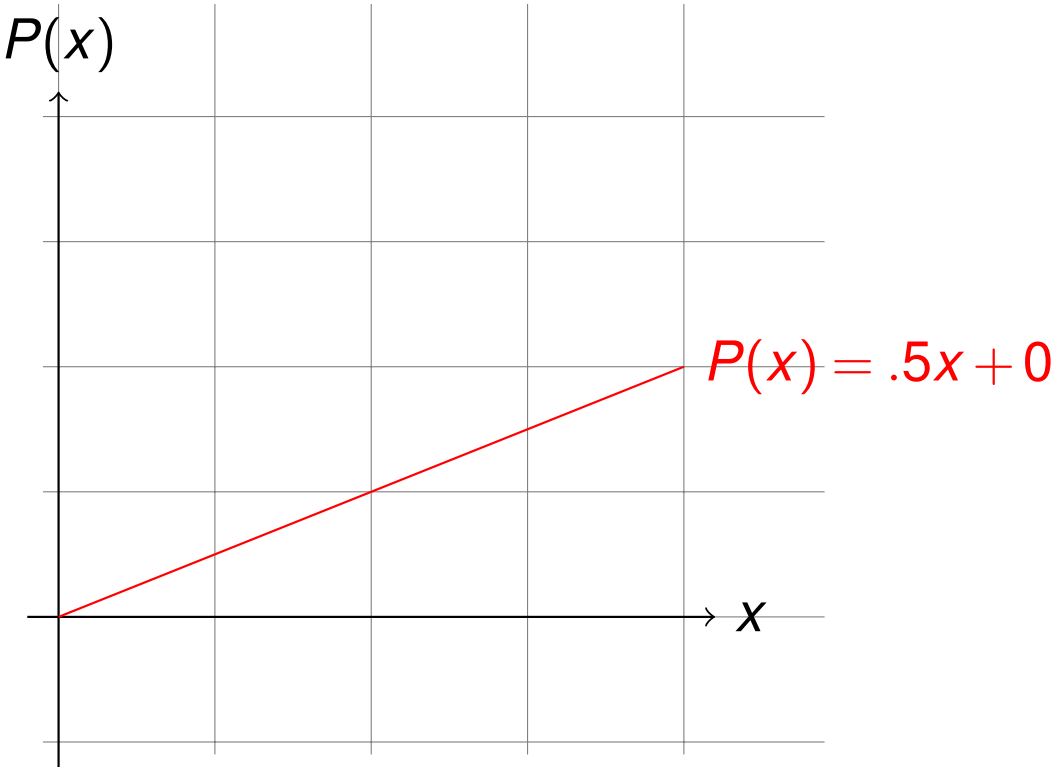

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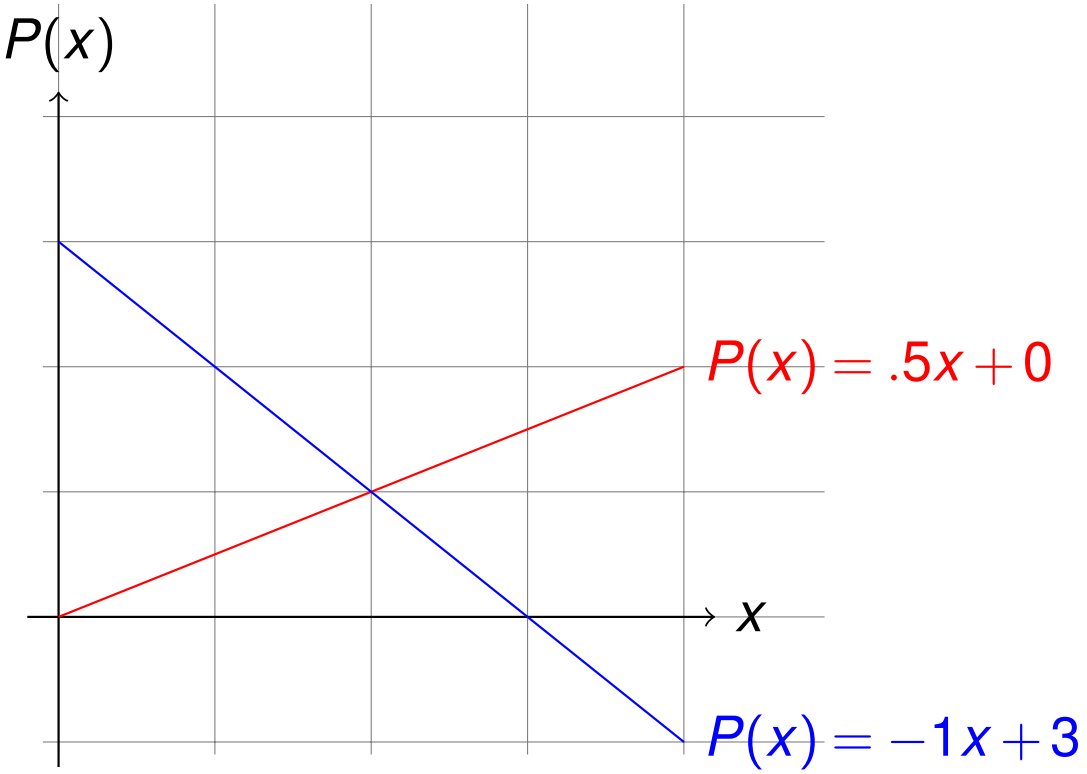

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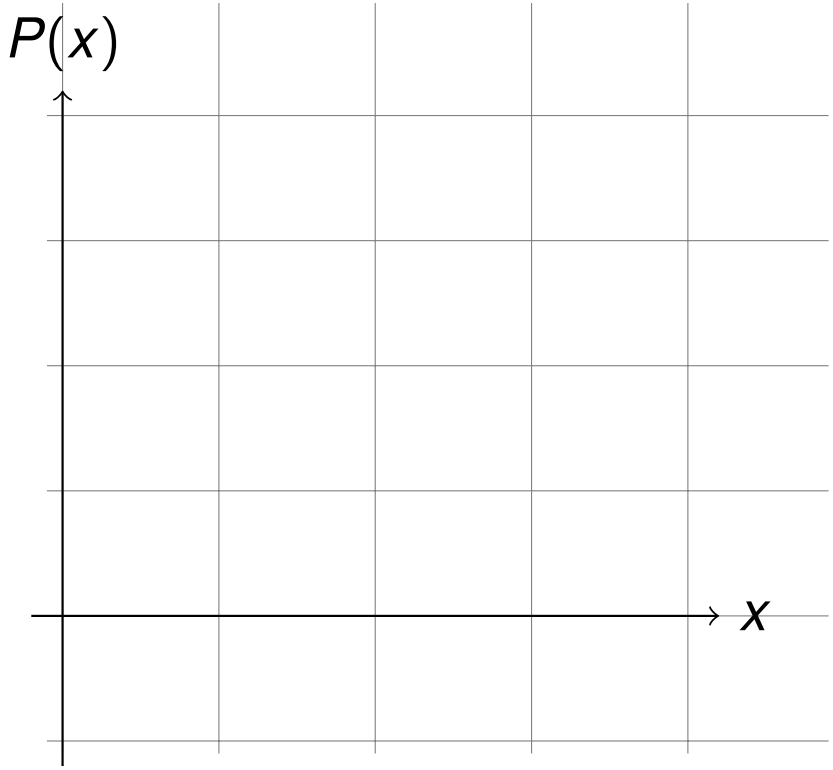

Parabola: $P(x) = a_2 x^2 + a_1 x + a_0$

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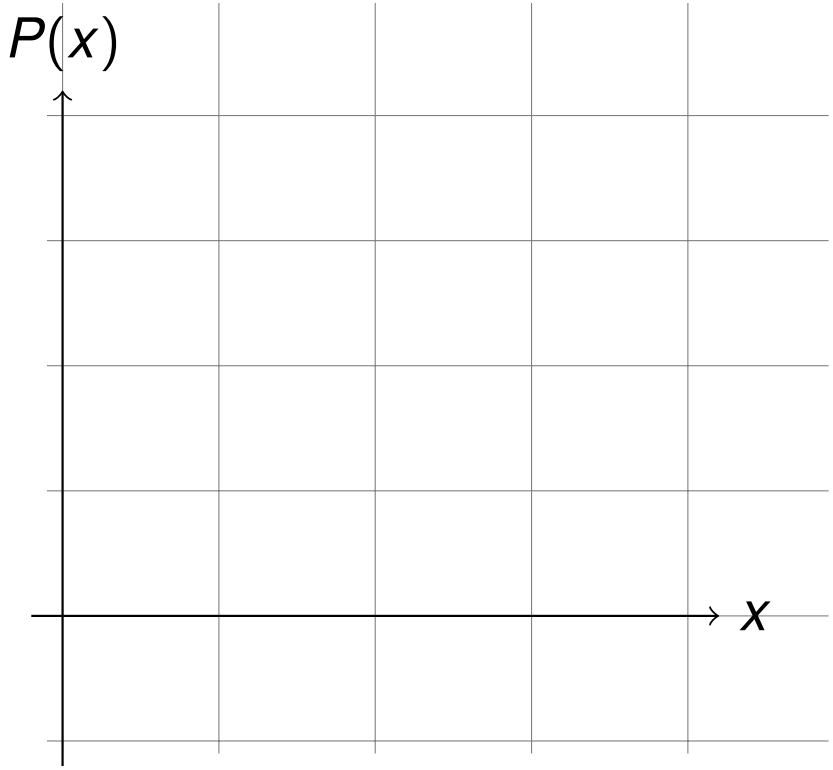

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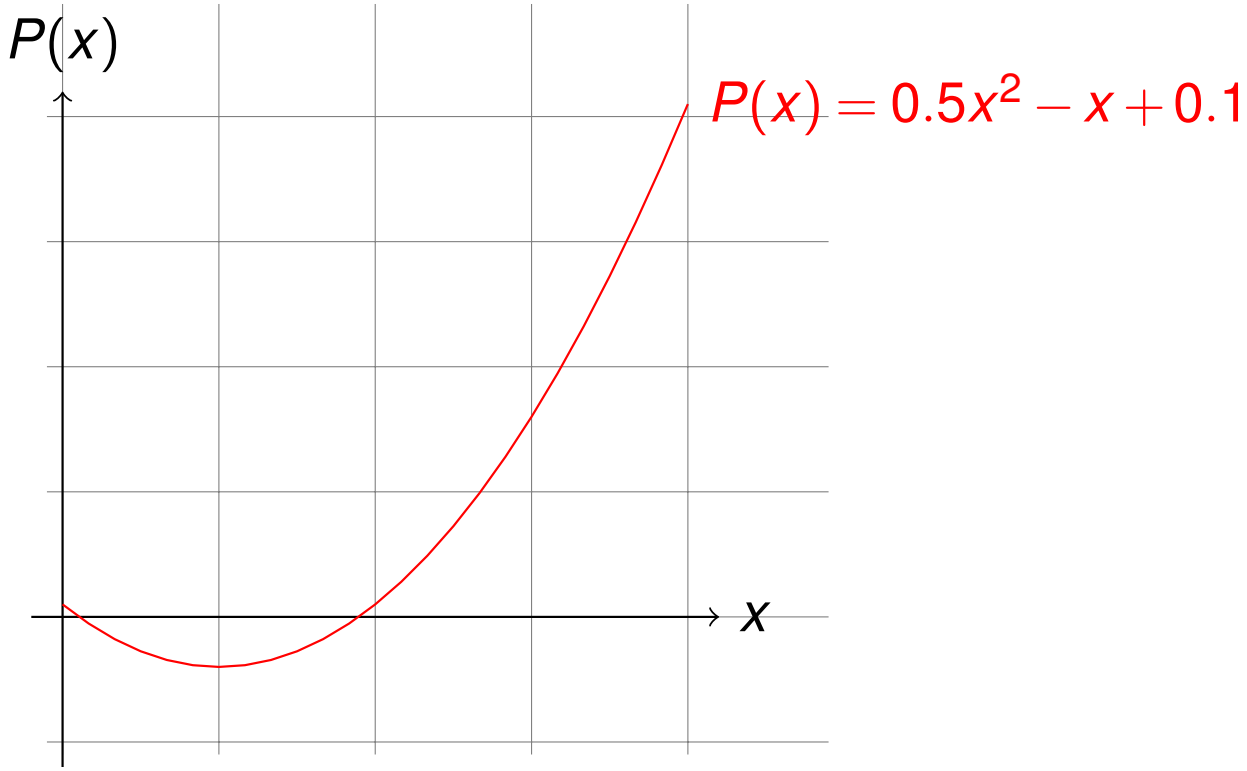

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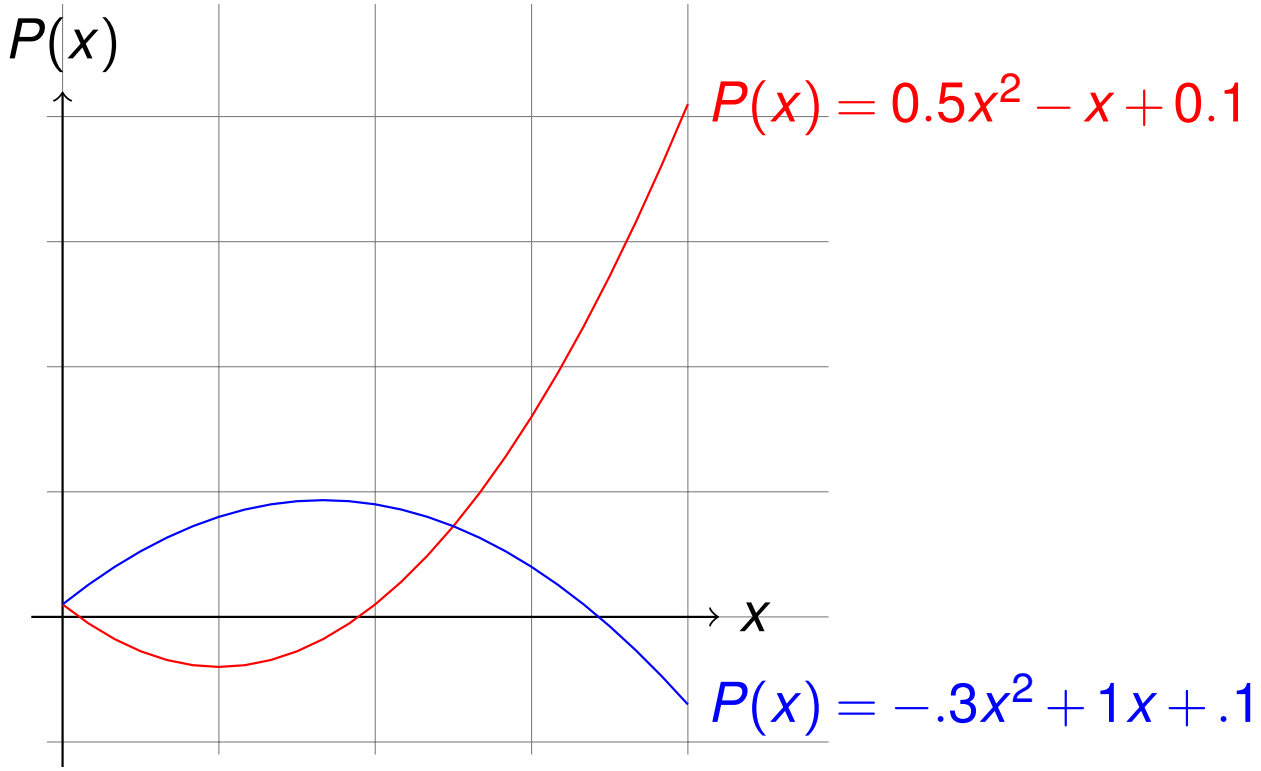

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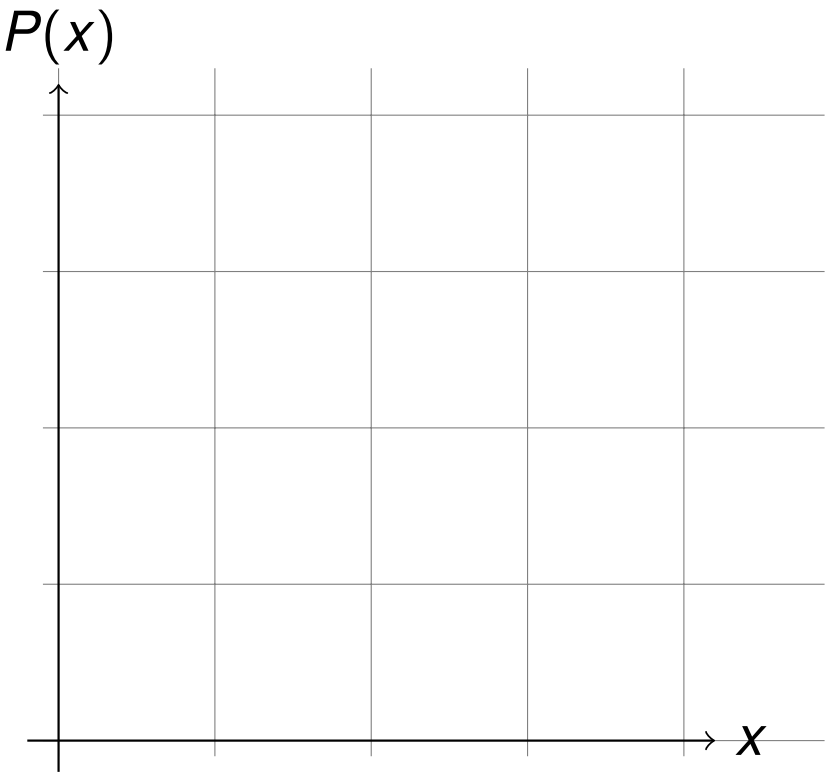

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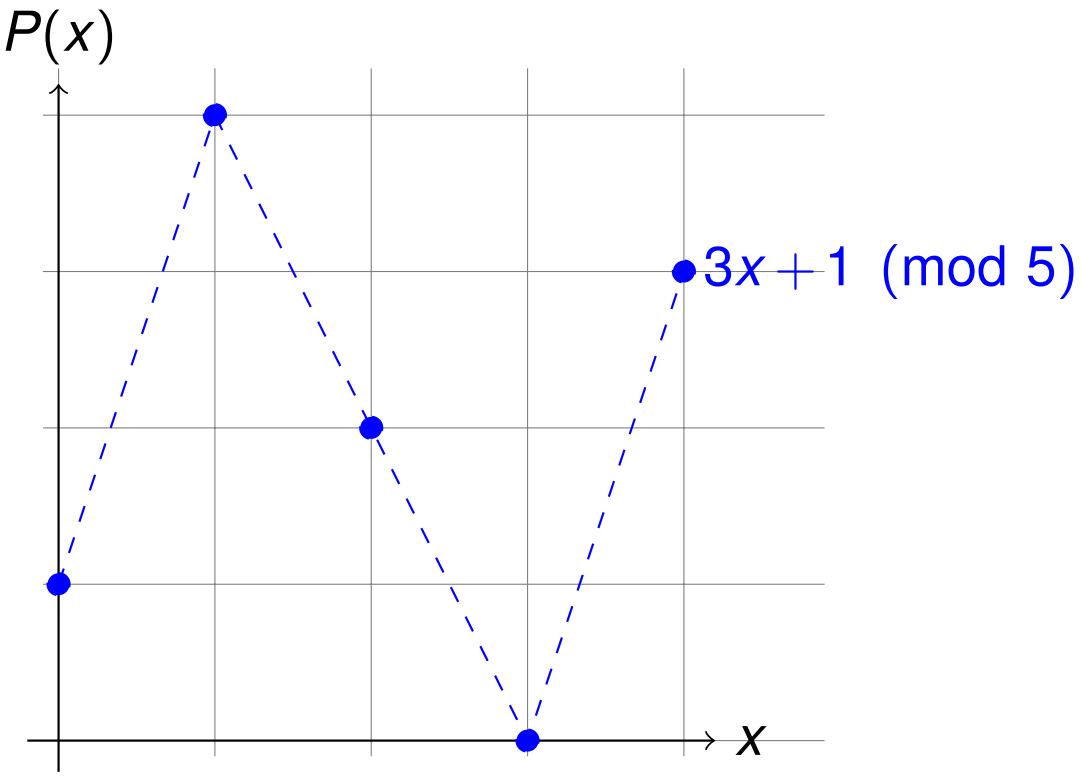

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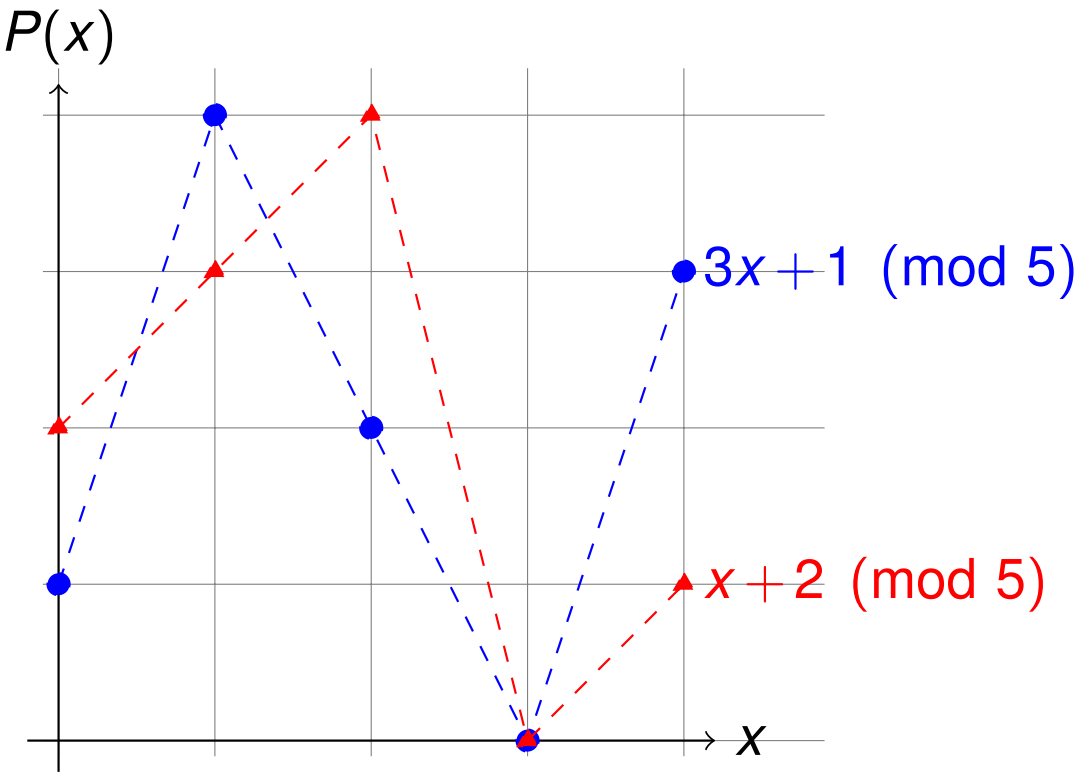

Finding an intersection.

$x+2 \equiv 3x+1 \pmod 5$

$\implies 2x \equiv 1 \pmod 5$

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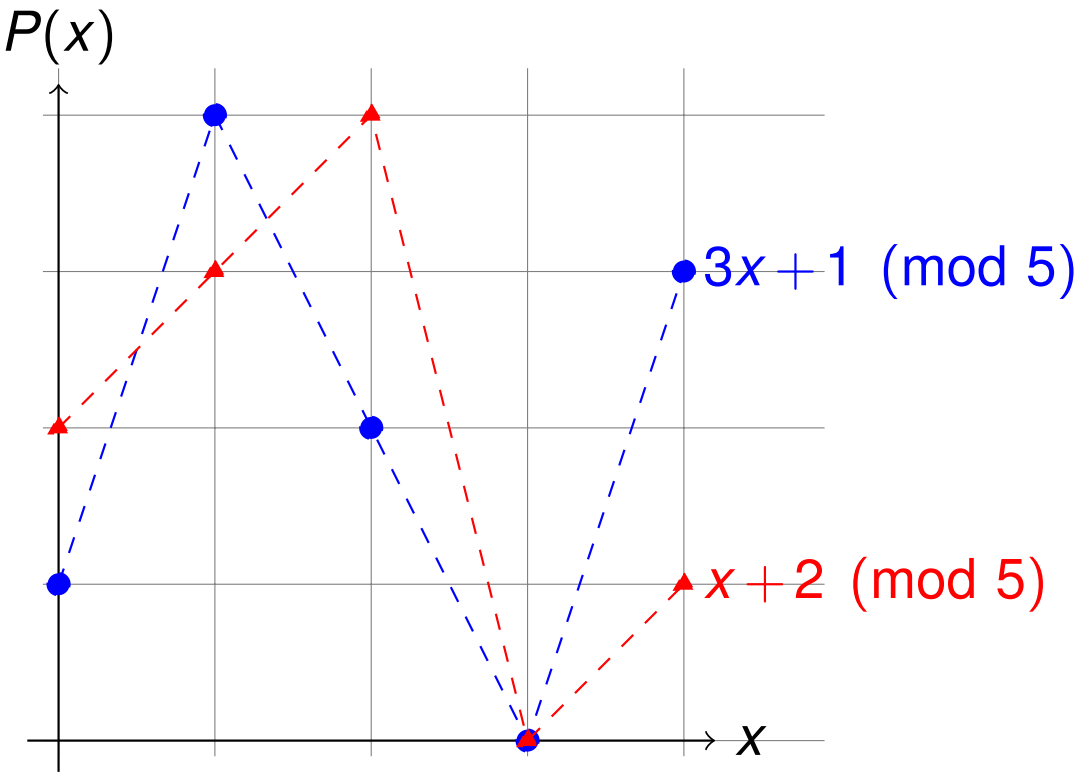

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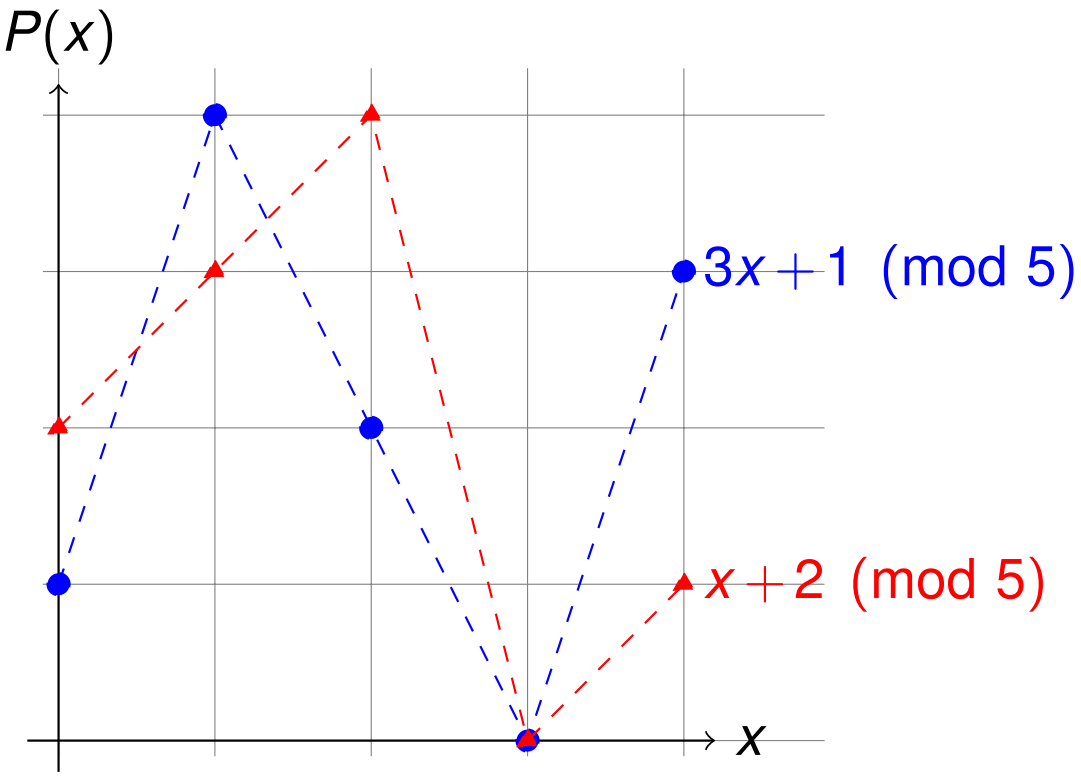

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Good when modulus is prime!!

Two points make a line.

Fact: Exactly 1 degree $\leq d$ polynomial contains $d+1$ points. [2]

[2] Points with different x values.

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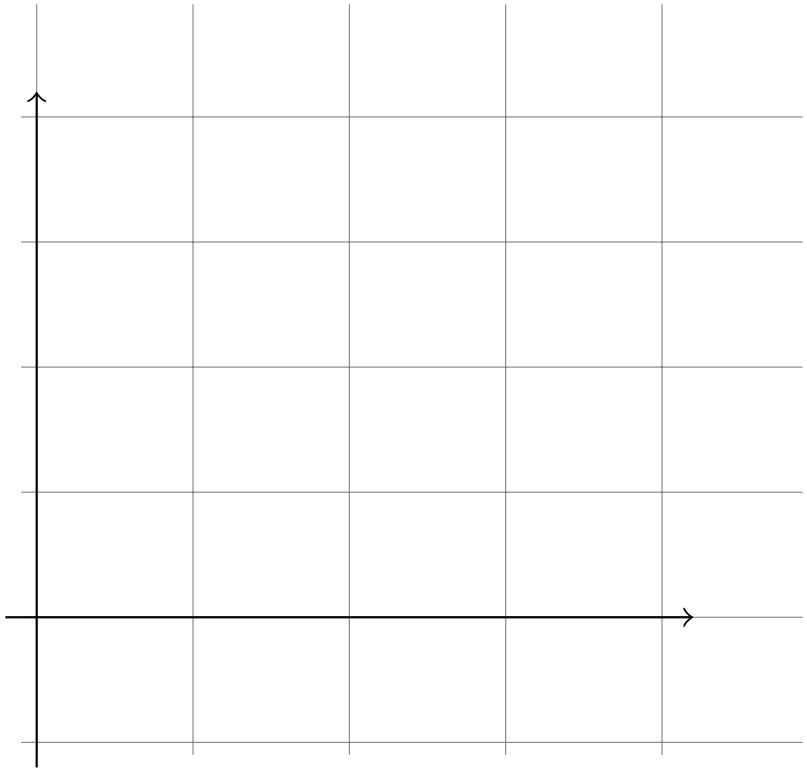
3 points determine a parabola.



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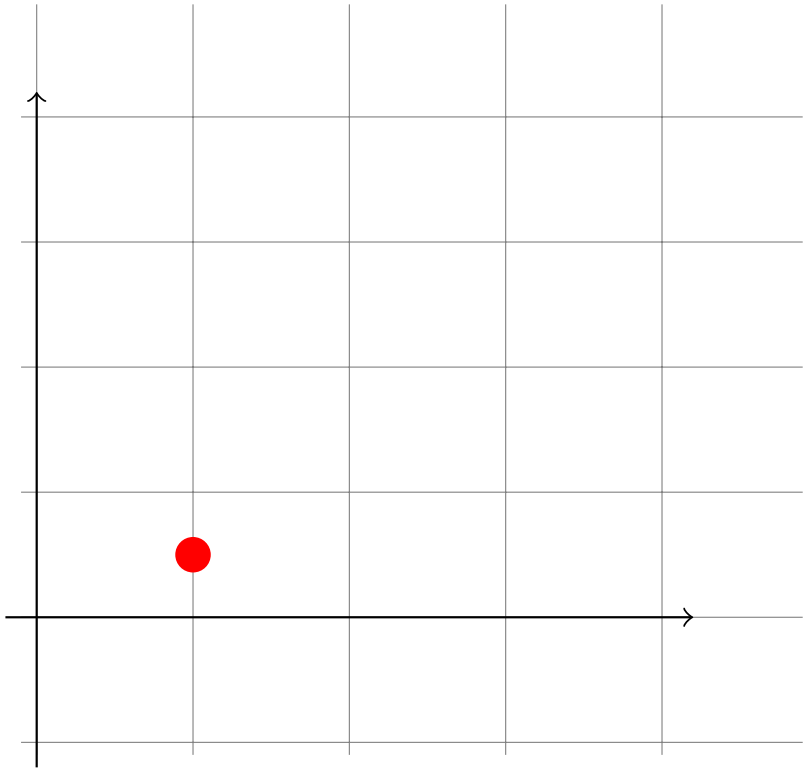

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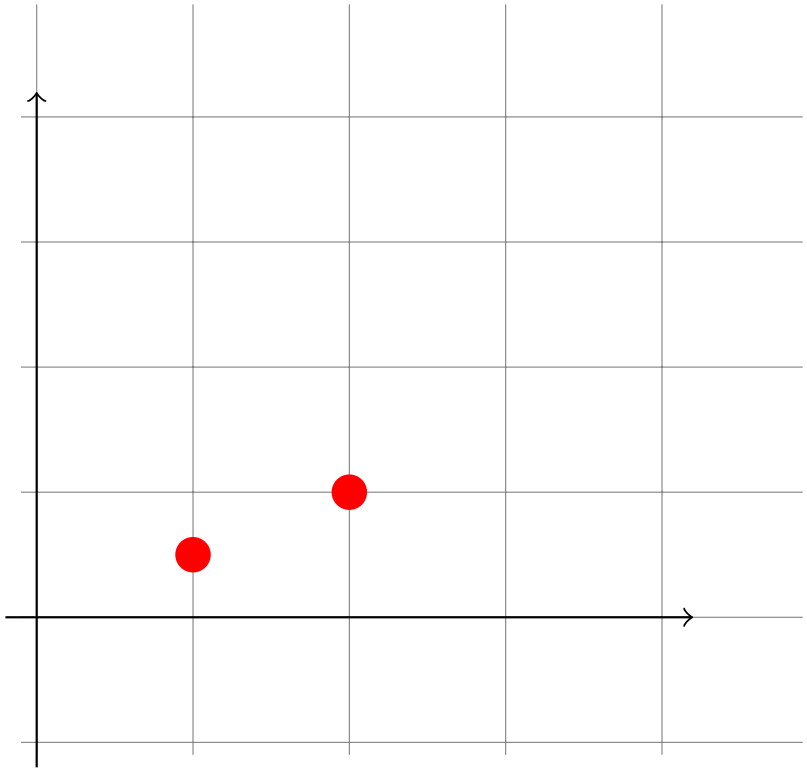

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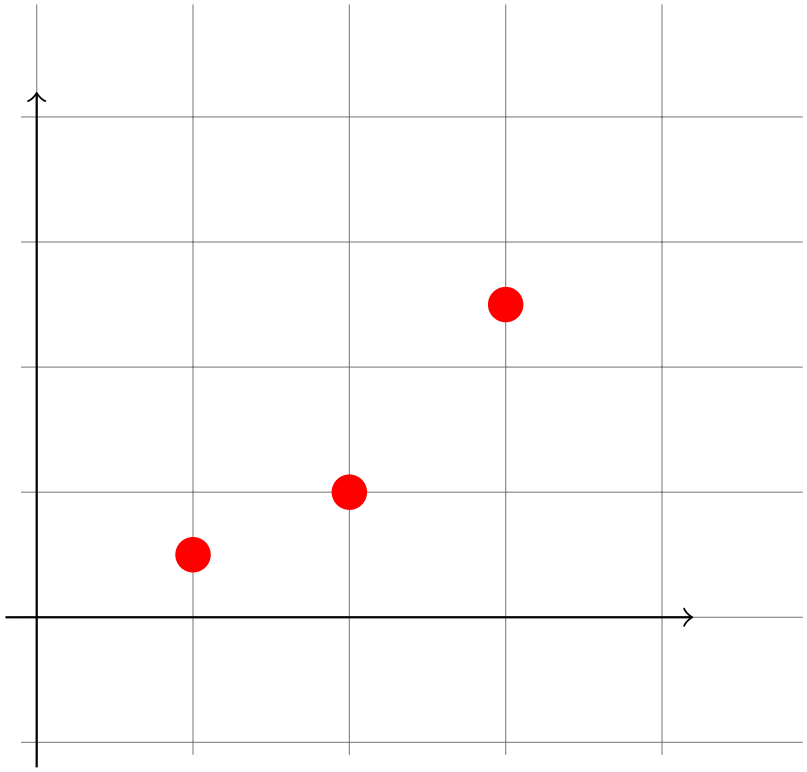

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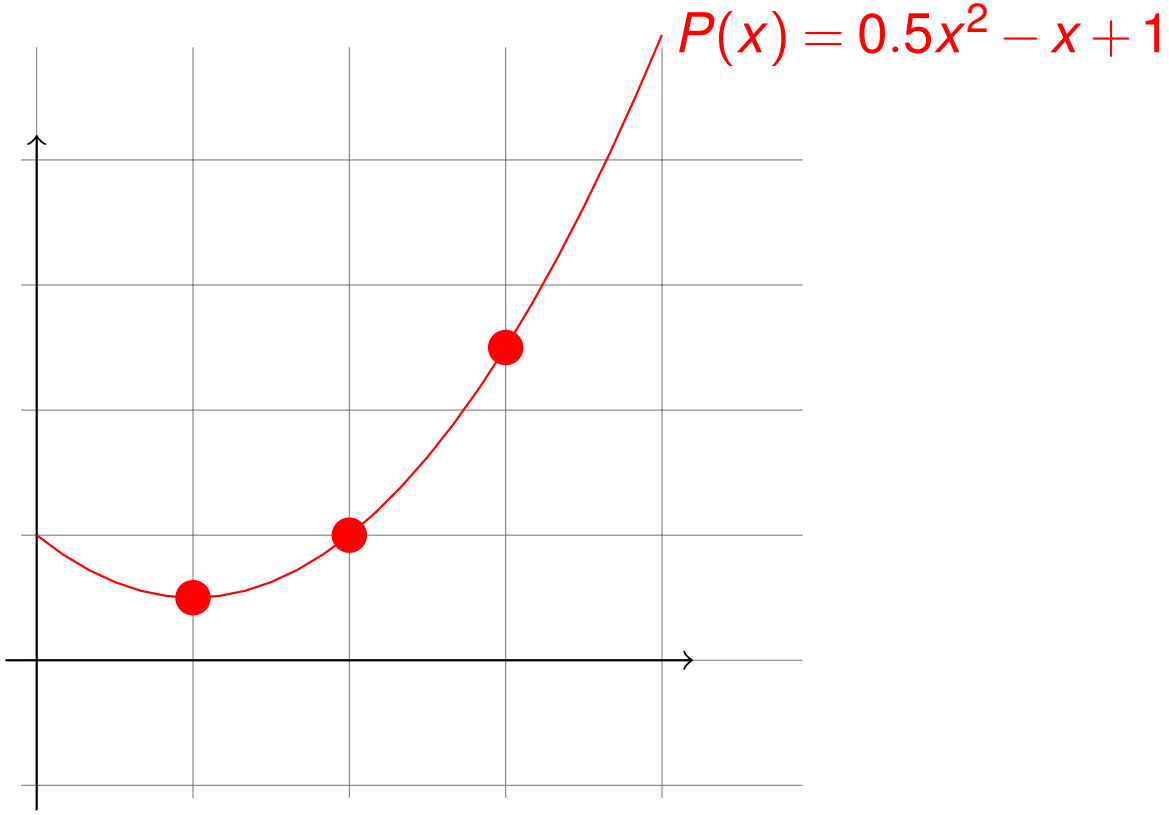

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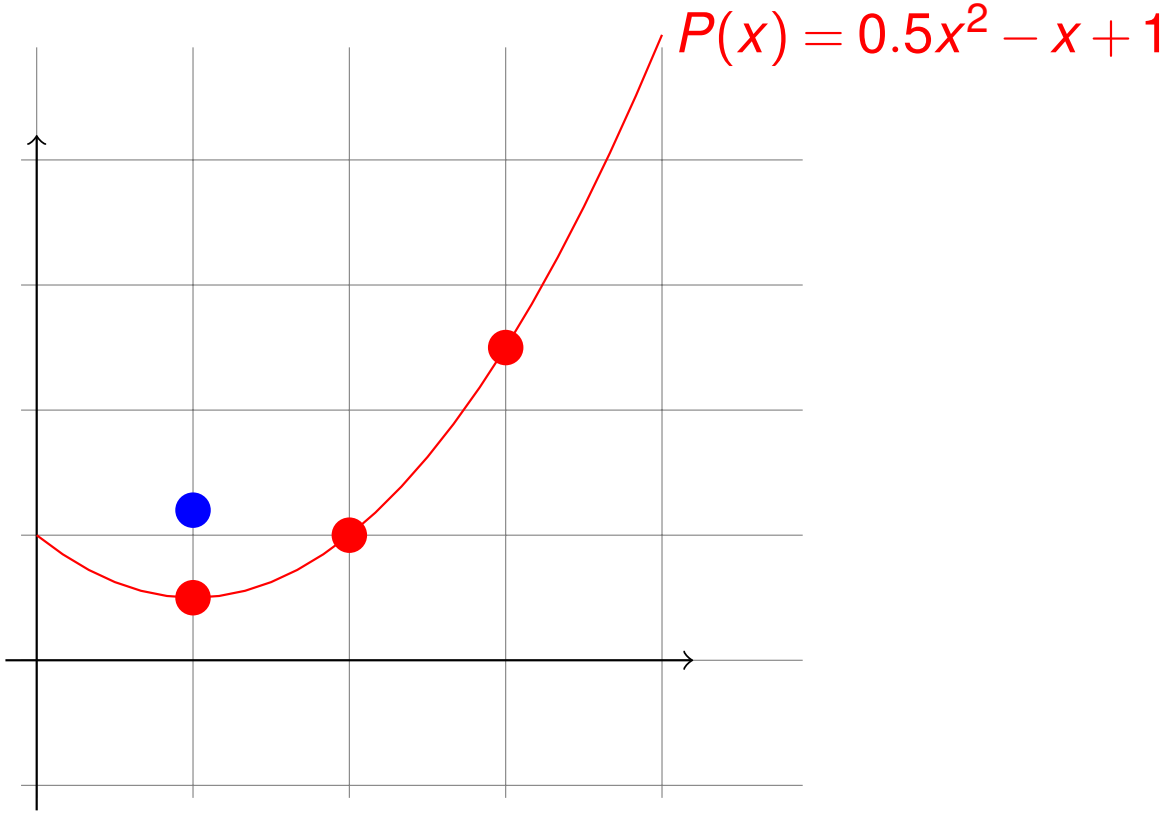

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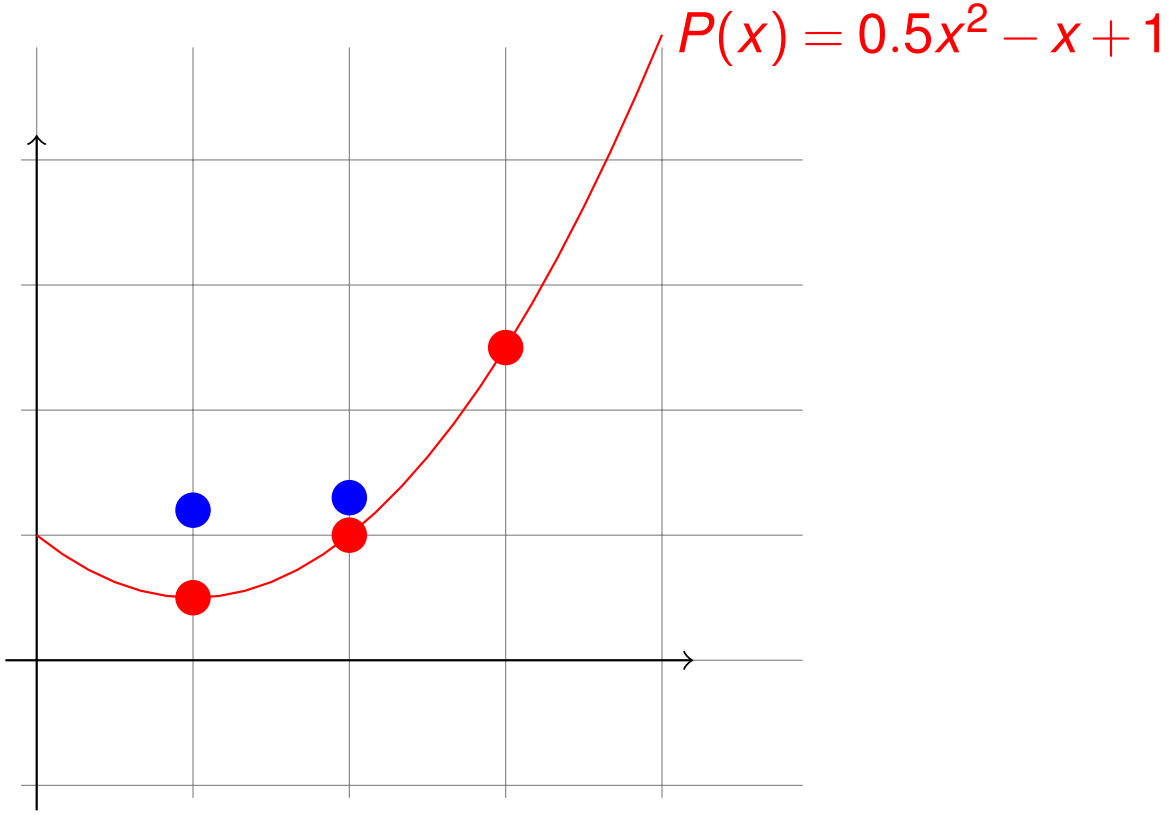

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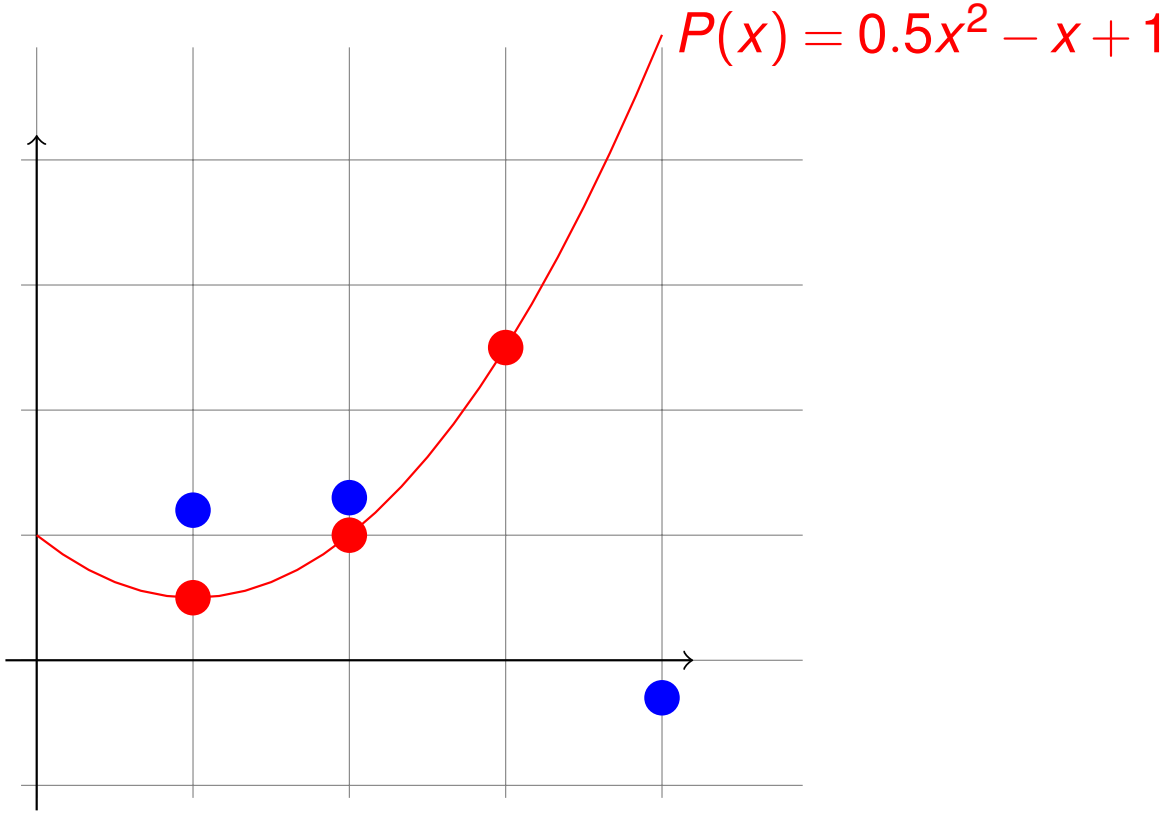

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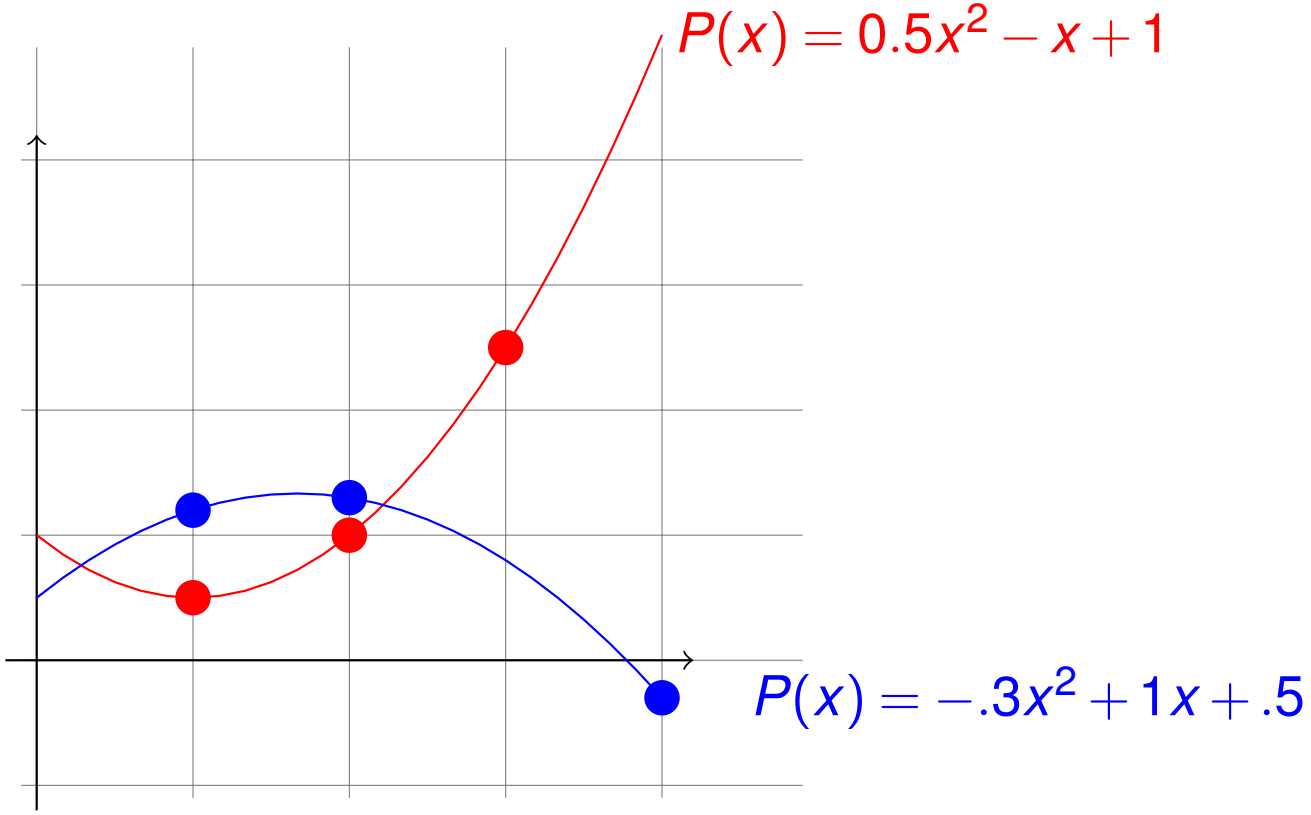

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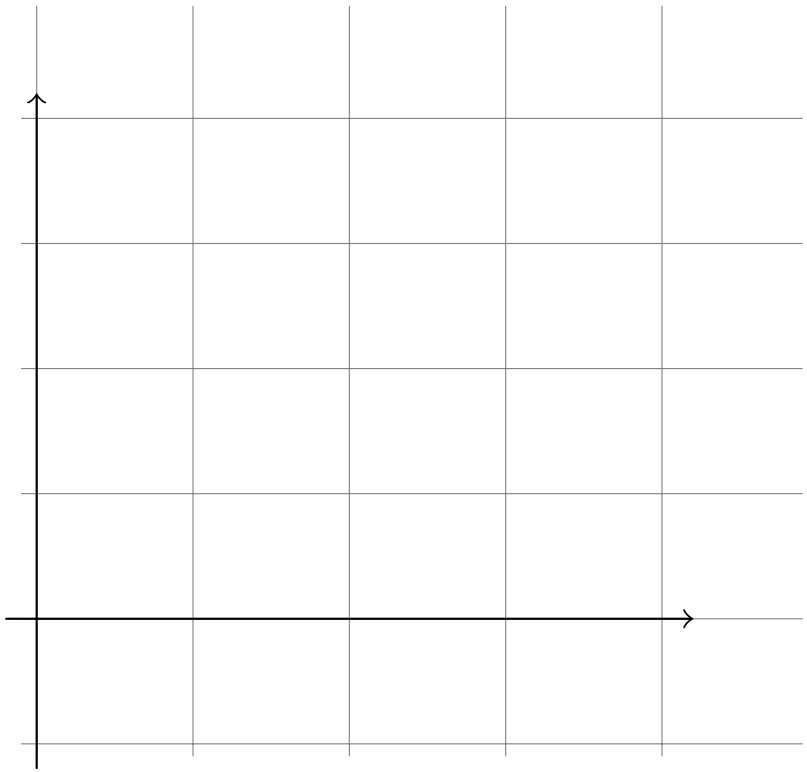

There is $P(x)$ contains blue points and *any* $(0, y)$!

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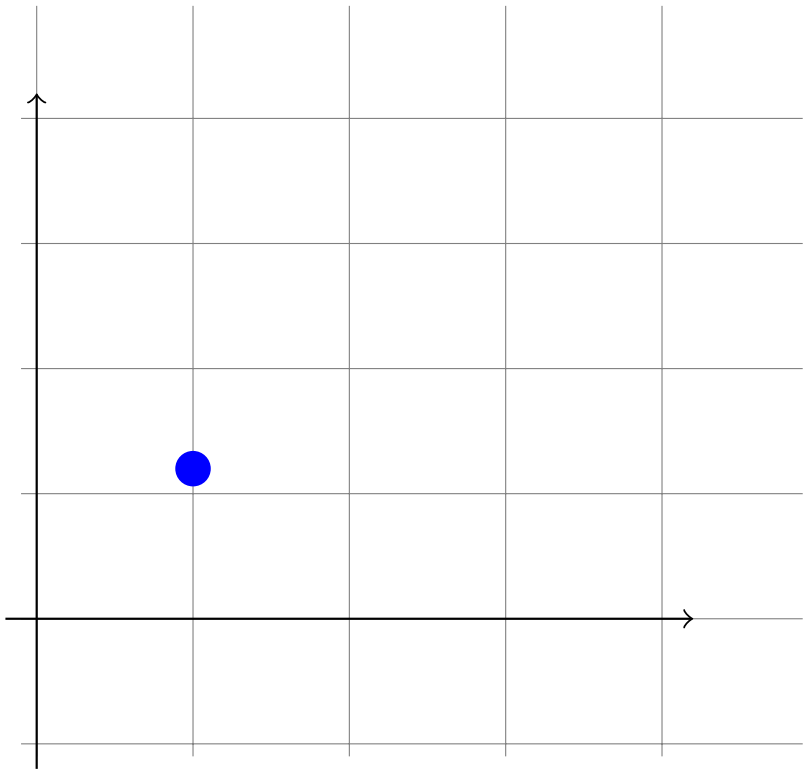

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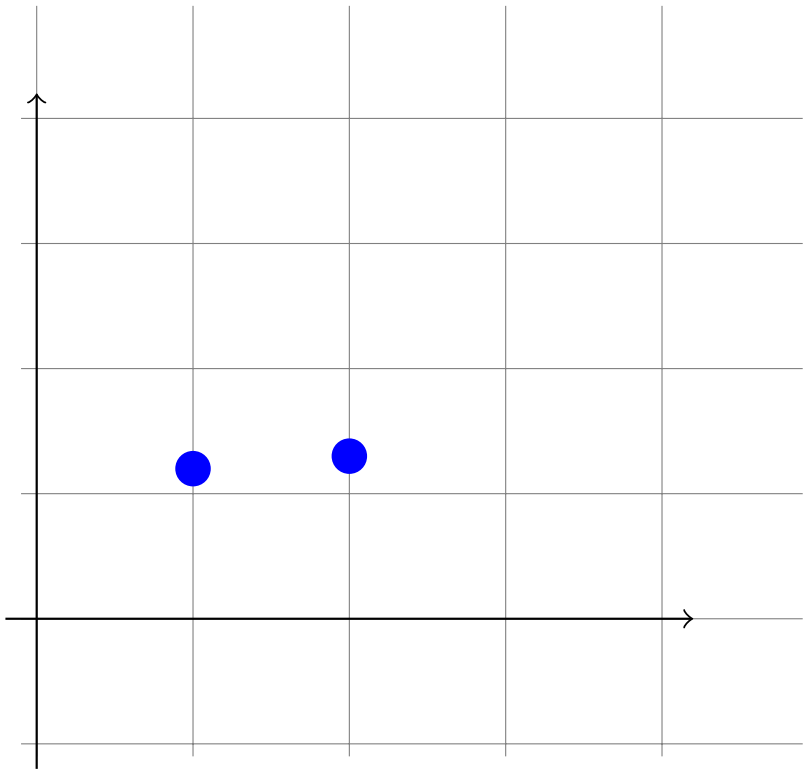

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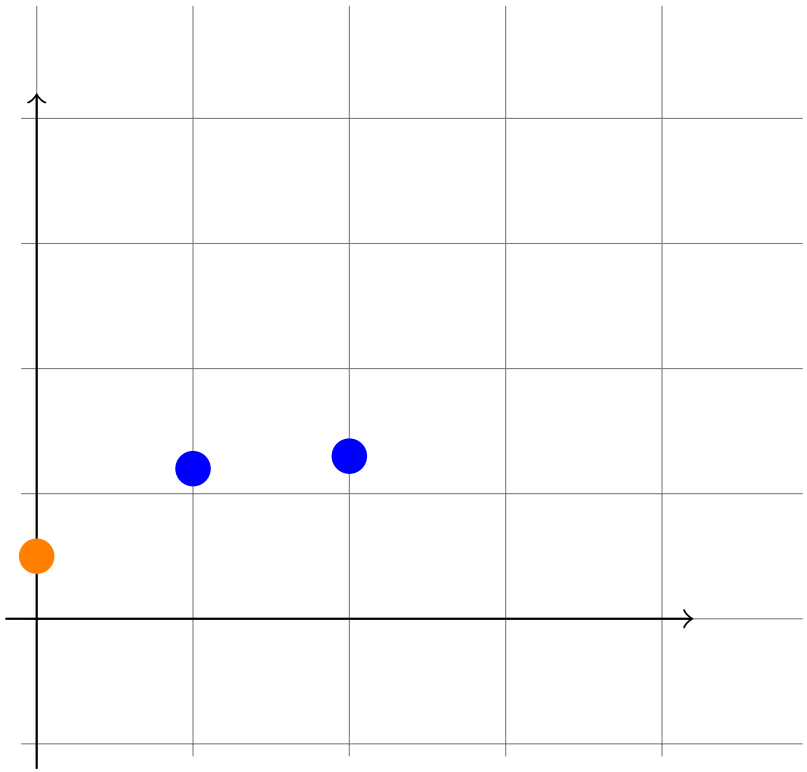

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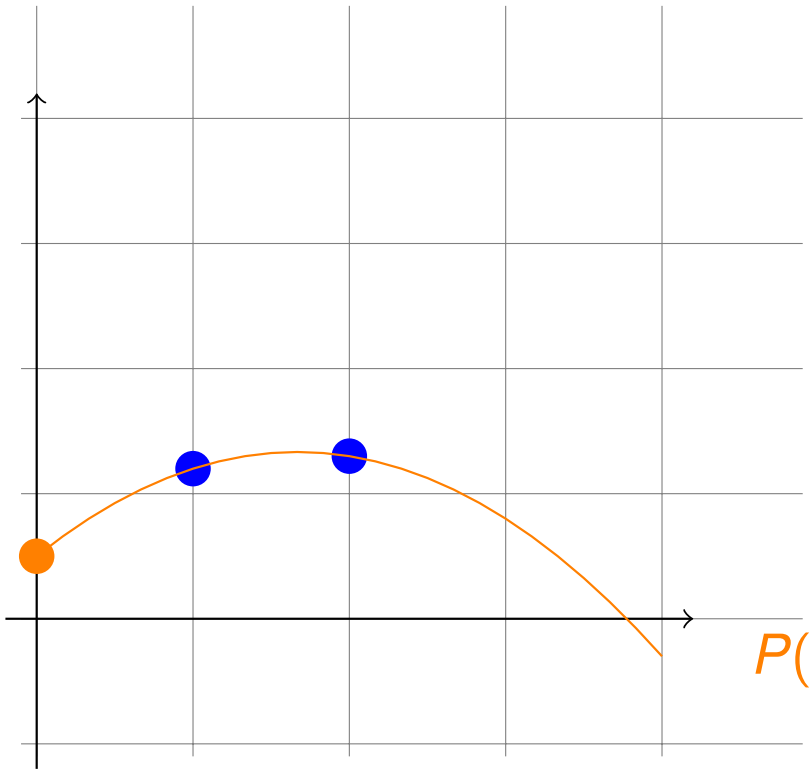

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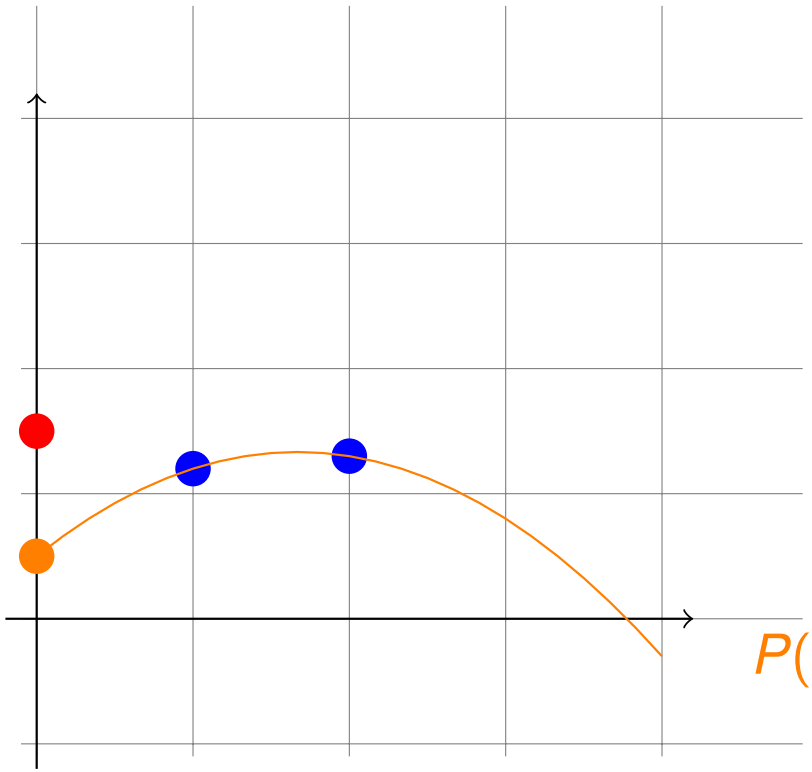

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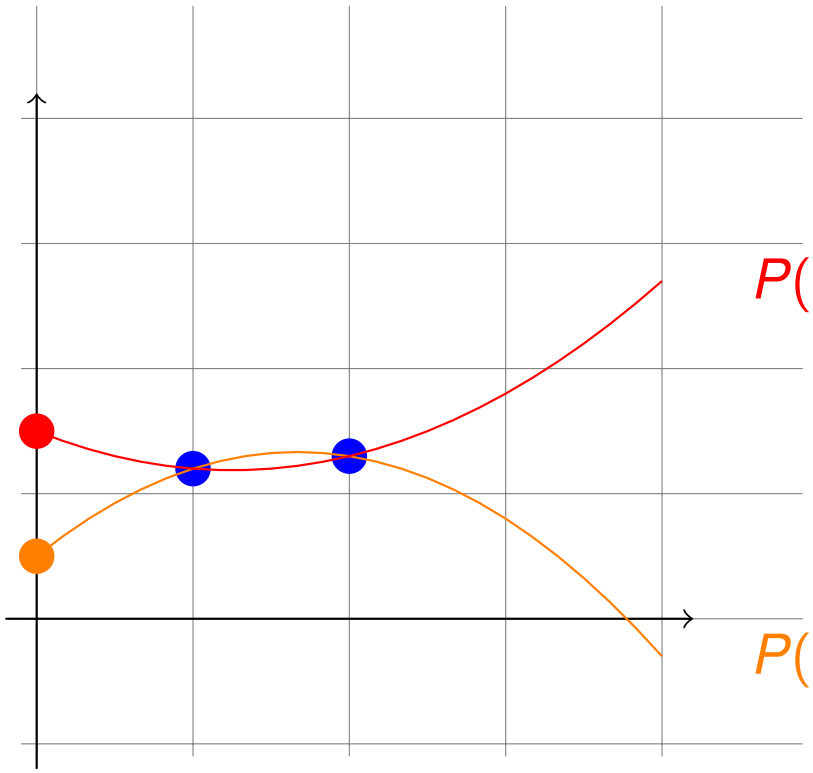

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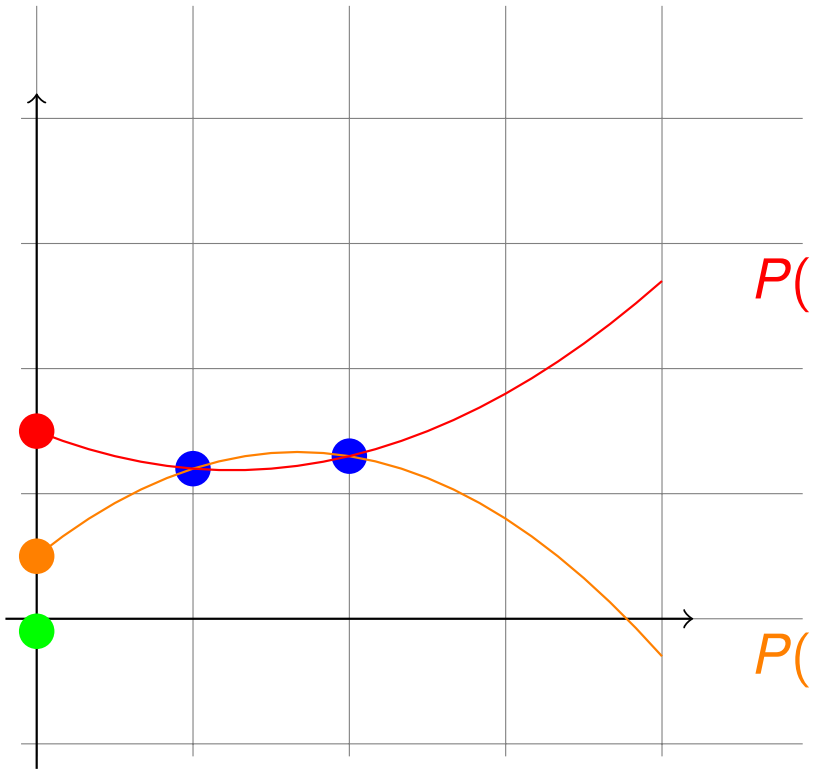

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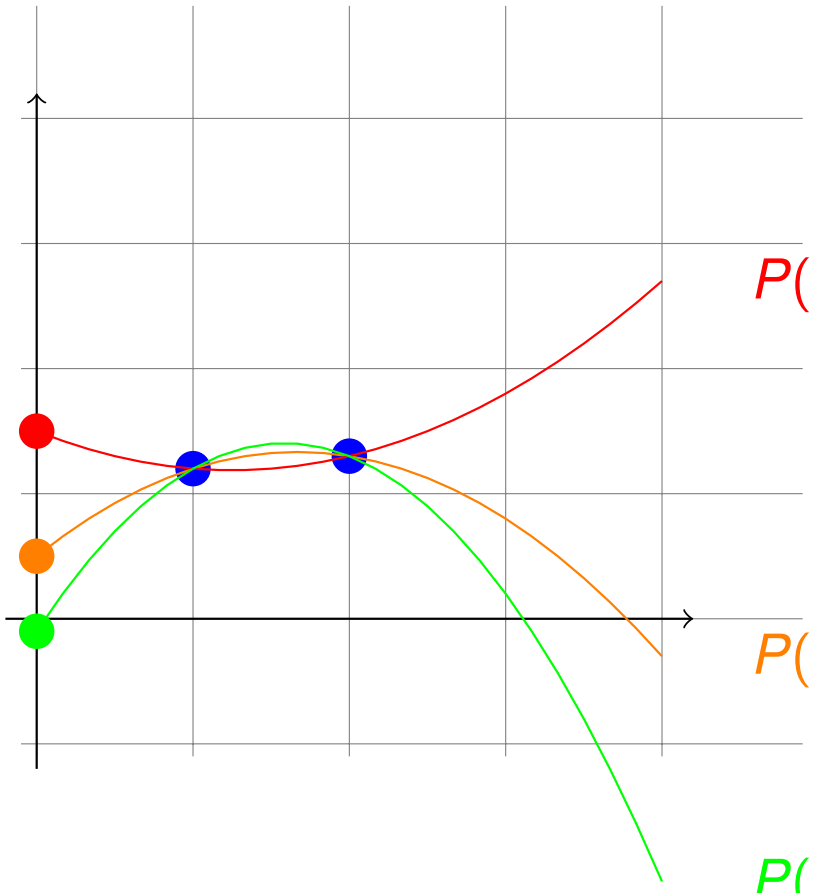

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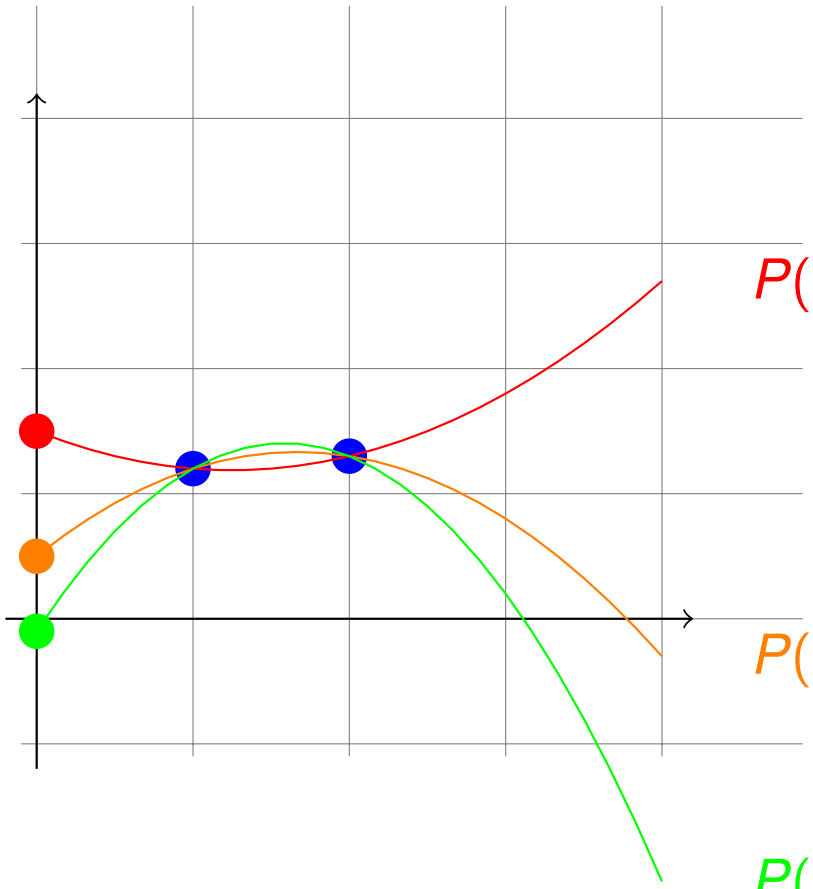

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For $k = 2$?

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Knowing k pts $\implies$ only one $P(x)$ $\implies$ evaluate $P(0)$.
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Modular Arithmetic Fact and Secrets

Modular Arithmetic Fact: Exactly 1 degree $\leq d$ polynomial with arithmetic modulo prime p contains $d+1$ pts.

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With only one point.

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Shares are $m(1) + b$, $m(2) + b$, $\ldots$
Any two determine line. And b.
With only one point. b can be anything!

We will work with polynomials with arithmetic modulo p.

Excursion.

Homework 4. Problem 2.

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$a = 1 \pmod 5$

Excursion.

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$a = 1 \pmod 5 \quad a = 0 \pmod 8$

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$$a = 1 \pmod 5 \qquad a = 0 \pmod 8$$
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$$c = 5 \pmod 5$$

Excursion.

Homework 4. Problem 2.

$$\begin{array}{ll} a = 1 \pmod 5 & a = 0 \pmod 8 \\ b = 0 \pmod 5 & b = 1 \pmod 8 \\ c = 5 \pmod 5 & c = 10 \pmod 8 \end{array}$$

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What is c in terms of a and b?

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$c = 5a +$

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What is c in terms of a and b?

$$c = 5a + 10b$$

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Idea: a and b, 1 for one modulus and 0 for (all) others.

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See Chinese Remainder Theorem: many moduli.

Delta Polynomials: Concept.

For set of x-values, $x_1, \ldots, x_{d+1}$.

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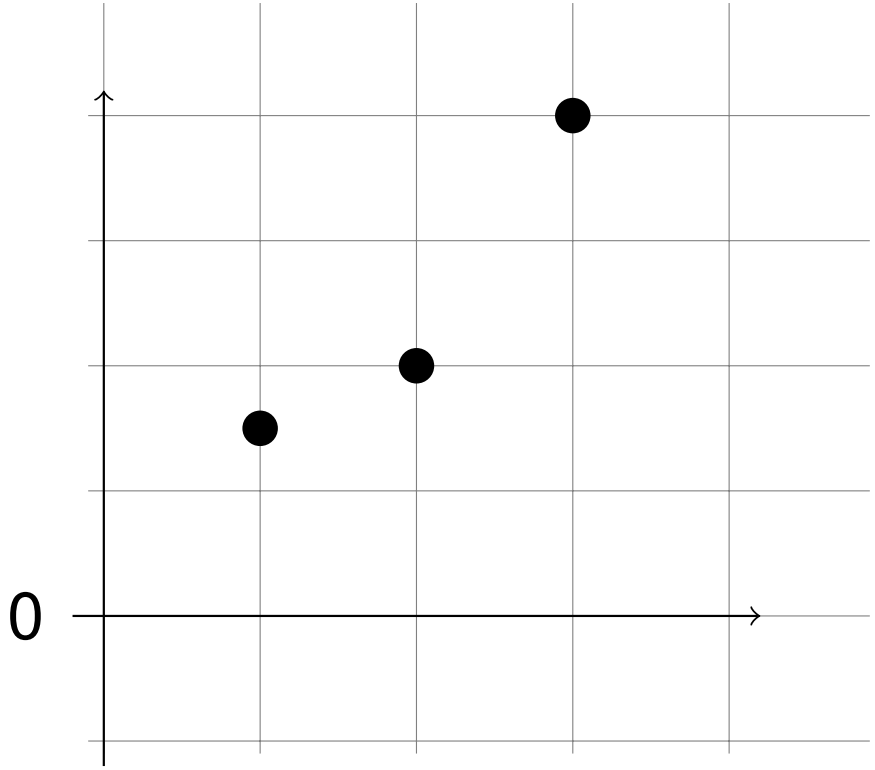
Delta functions: in pictures.

Points: $(1, 1.5)$, $(2, 2)$, $(3, 4)$.



Scale each Δ_i function and add to contain points.

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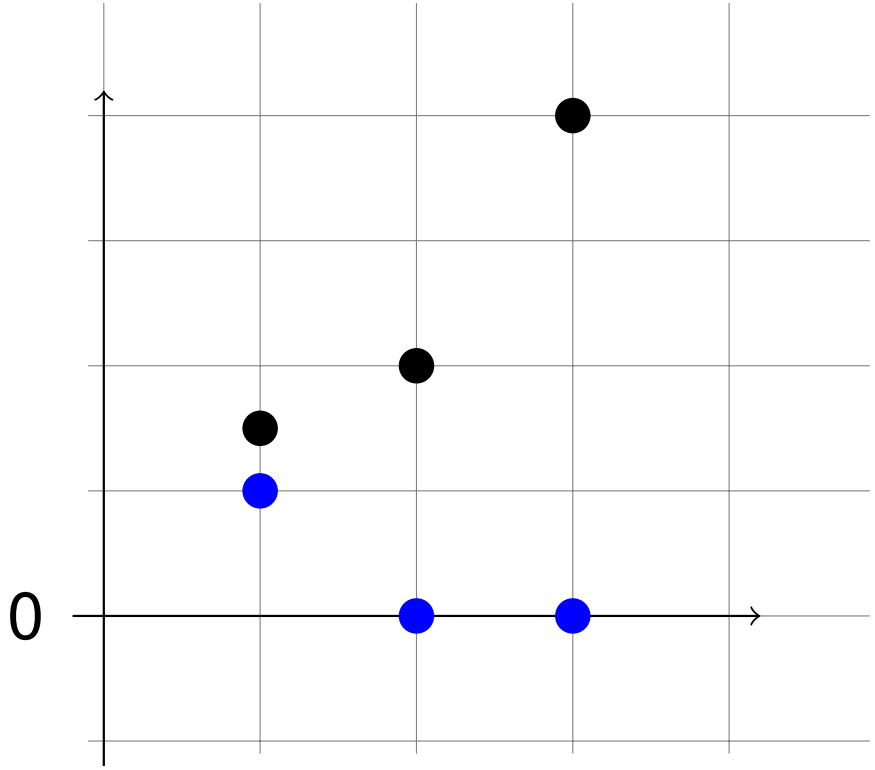

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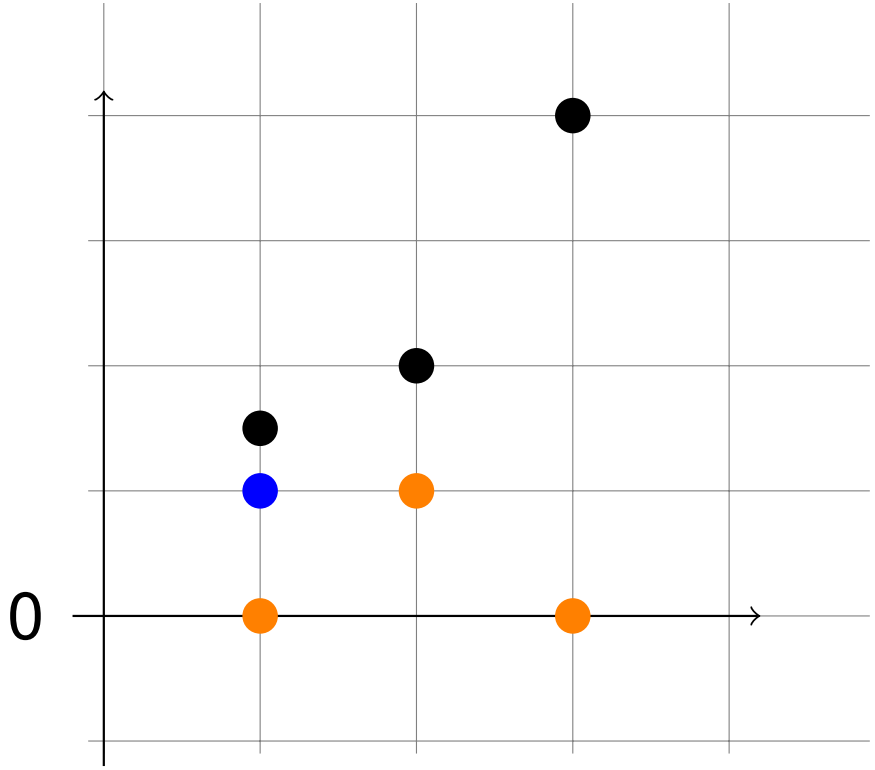

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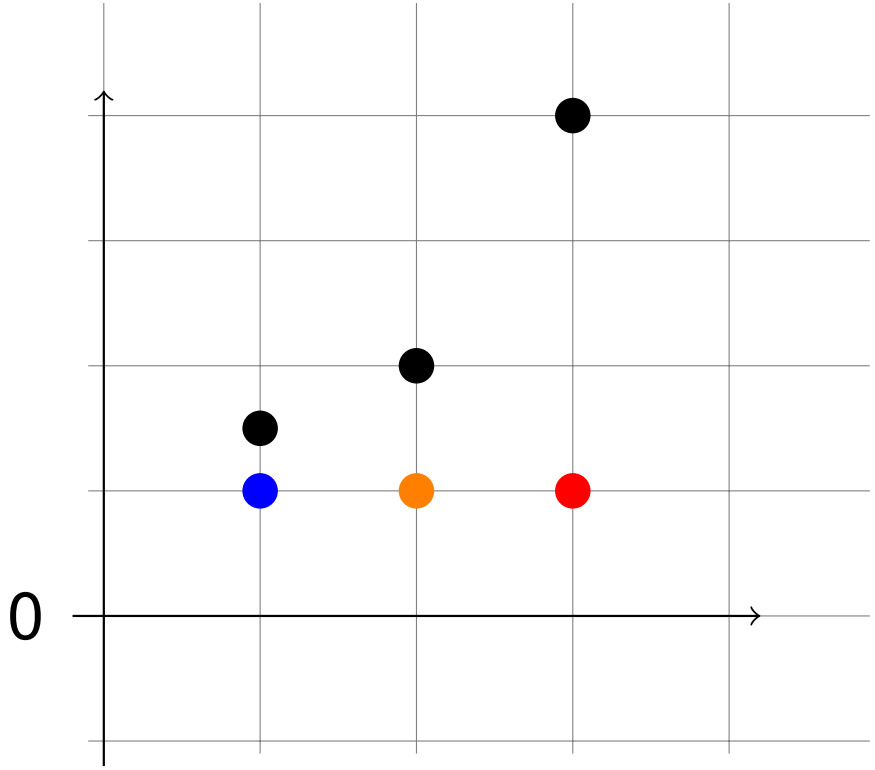

Delta functions: in pictures.

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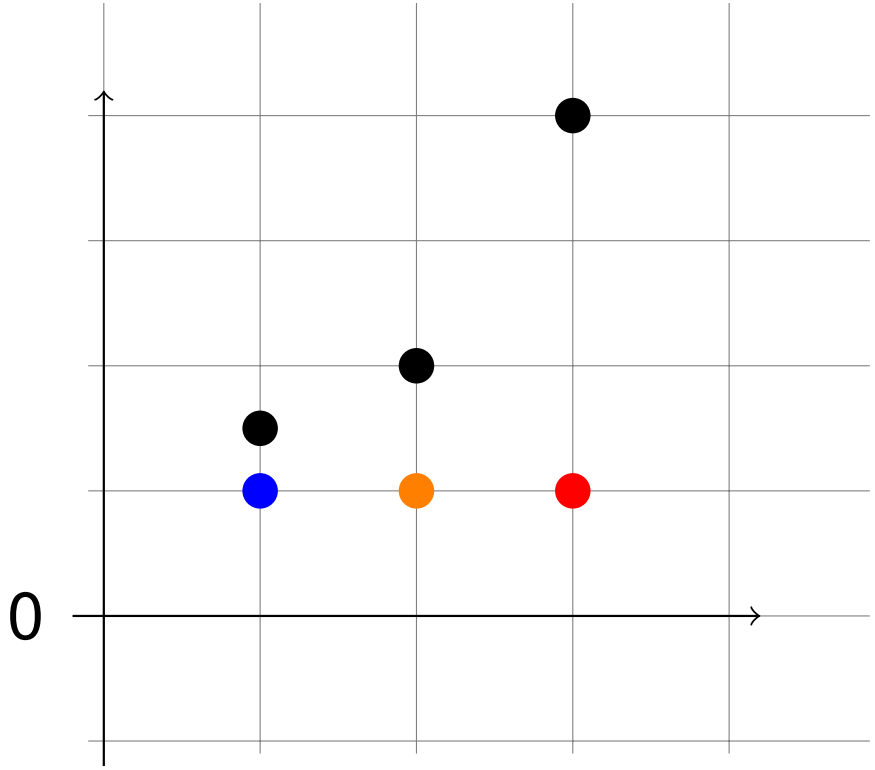

Scale each Δ_i function and add to contain points.

Delta functions and polynomials.

Degree 1 polynomial, $P(x)$, that contains $(1,3)$ and $(3,4)$?

Delta functions and polynomials.

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Work modulo 5.

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$\Delta_1(x)$?

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$\Delta_1(1) = 1$

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Divide by 3!

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The Construction: Interpolation!

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Construction proves the existence of the polynomial!

For secret sharing.

Modular Arithmetic Fact: Exactly 1 degree $\leq d$ polynomial with arithmetic modulo prime p contains $d+1$ pts.

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Wednesday!