Problem: Want to send a message with n packets.

Channel: Lossy channel: loses k packets.

Question: Can you send $n+k$ packets and recover message?

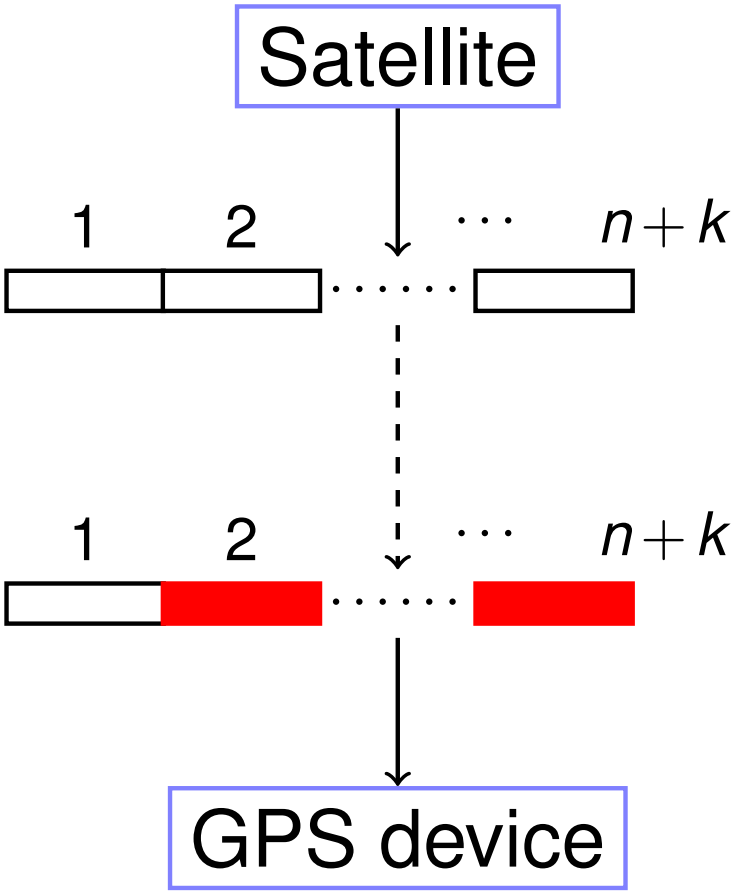
A degree $n-1$ polynomial determined by any n points!

Erasure Coding Scheme: message = $m_0, m_2, \ldots, m_{n-1}$.

1. Choose prime $p \approx 2^b$ for packet size b.
2. $P(x) = m_{n-1}x^{n-1} + \cdots m_0 \pmod{p}$.
3. Send $P(1), \ldots, P(n+k)$.

Any n of the $n+k$ packets gives polynomial ...and message!

Erasure Codes.



n packet message. So send $n+k$!

Lose k packets.

Any n packets is enough!

n packet message.

Optimal.

Complexity Issues.

Size: Can choose a prime between 2^{b-1} and 2^b.
(Lose at most 1 bit per packet.)

In practice, $O(n)$ operations with almost the same redundancy.

Polynomials.

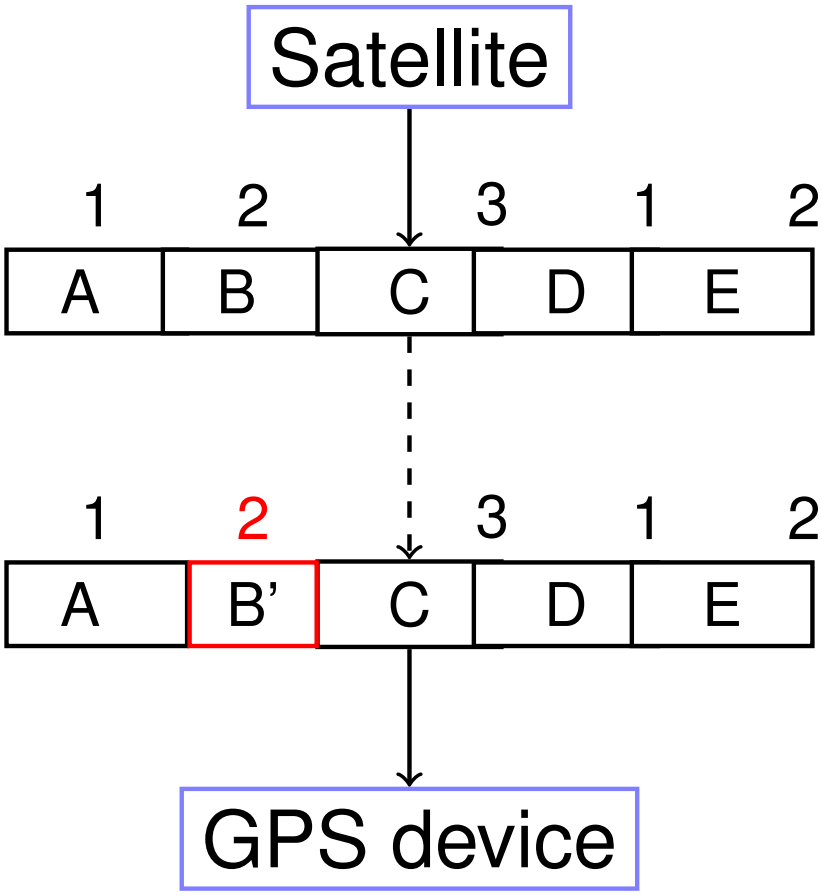
- ..give Secret Sharing.
- ..give Erasure Codes.

Error Correction:

Noisy Channel: corrupts k packets. (rather than loses.)

Additional Challenge: Finding which packets are corrupt.

Error Correction



3 packet message. Send 5.

Corrupts 1 packets.

The Scheme.

Problem: Communicate n packets $m_1, \ldots, m_n$ on noisy channel that corrupts $\leq k$ packets.

Reed-Solomon Code:

1. Make a polynomial, $P(x)$ of degree $n-1$, that encodes message.
   - $P(1) = m_1, \ldots, P(n) = m_n$.
   - Comment: could encode with packets as coefficients.
2. Send $P(1), \ldots, P(n+2k)$.

After noisy channel: Recieve values $R(1), \ldots, R(n+2k)$.

Property: $P(i) = R(i)$ for at least $n+k$ points i.

Slow solution.

Brute Force: For each subset of $n+k$ points fit degree $n-1$ polynomial to them.

- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!
- For any subset of $n+k$ pts,
  1. at least n pts are correct since only k errors,
  2. $P(x)$ is only degree $n-1$ polynomial that contains the n correct points.!

Reconstructs $P(x)$ and only $P(x)$!!

Details..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(n+2k)$.

$$\begin{aligned}
p_{n-1} + \cdots p_0 &\equiv R(1) \pmod{p} \\
p_{n-1}2^{n-1} + \cdots p_0 &\equiv R(2) \pmod{p} \\
&. \\
p_{n-1}i^{n-1} + \cdots p_0 &\equiv R(i) \pmod{p} \\
&. \\
p_{n-1}(n+2k)^{n-1} + \cdots p_0 &\equiv R(n+2k) \pmod{p}
\end{aligned}$$

Error!! .... Where???
Could be anwhere!!! ...so try everything.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!.

Reed Solomon codes of size $n+2k$ can tolerate k errors!

The scheme is information optimal:
Must send $n+2k$ packets to recover from k errors.

- Any two codewords must be different by $2k$.
- Total information must equal original n packets, plus which k packets are corrupted.
  $n+2k$ packets, k garbage packets: $n+k$ packets of information.

Runtime is bad: exponential in k.

Find the errors faster?

...on Wednesday.