Error Correction

Satellite

GPS device

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3 packet message.

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Corrupts 1 packets.

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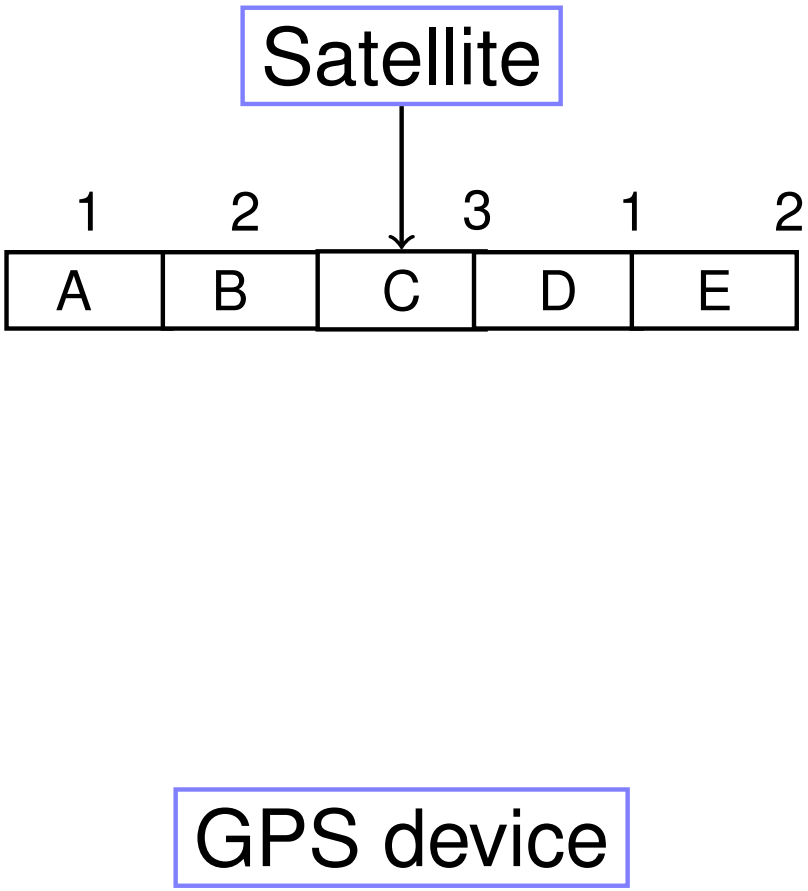

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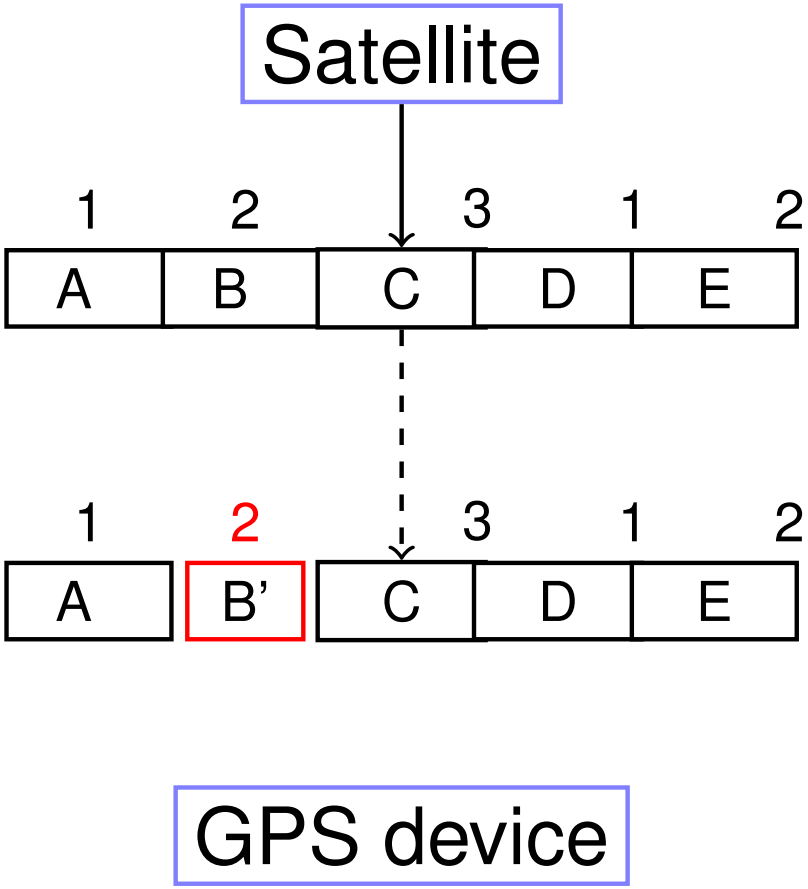

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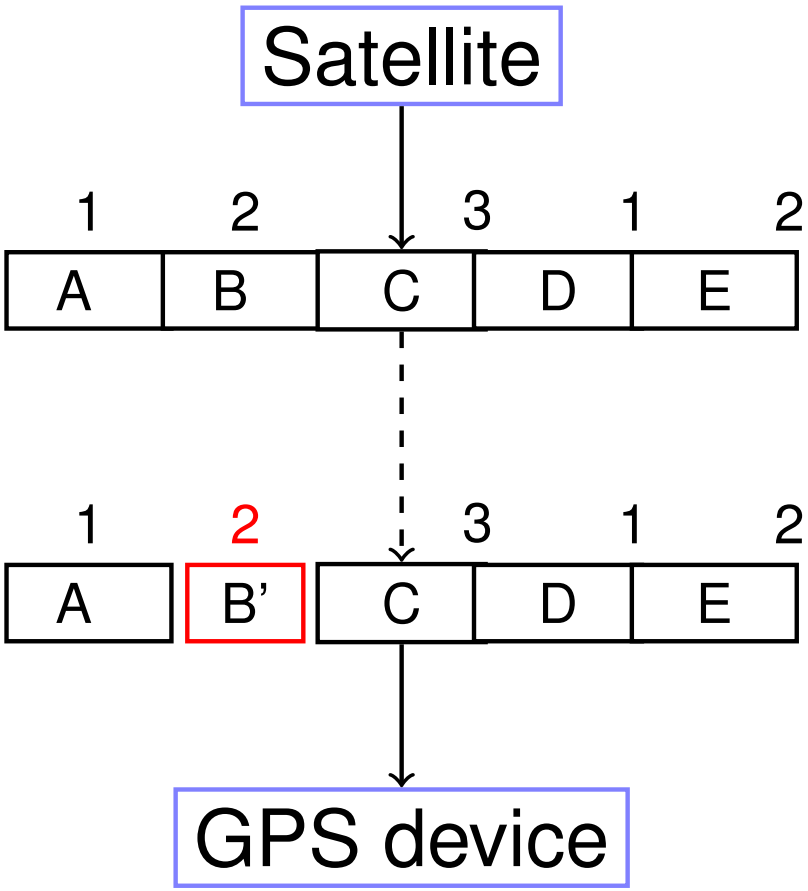

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$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

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If yes, output $Q(x)$.

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Reconstructs $P(x)$ and only $P(x)$!!

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Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

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All equations..

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\begin{aligned}
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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

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\begin{aligned}
p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\
4p_2 + 2p_1 + p_0 &\equiv 1 \pmod 7 \\
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Assume point 1 is wrong and solve..no consistent solution!

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In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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Could be anywhere!!! ...so try everywhere.

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Runtime: $\binom{n+2k}{k}$ possibilitities.

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Something like $(n/k)^k$ ...Exponential in k!.

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How do we find where the bad packets are efficiently?!?!?!

Ditty...

Ditty...

Where oh where

Ditty...

Where oh where can my bad packets be ...

Ditty...

Where oh where can my bad packets be ...

Ditty...

Where oh where can my bad packets be ...

Today.

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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\end{aligned}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Where oh where can my bad packets be?

$$
\begin{array}{rrcll}
 & (p_{n-1} + \cdots p_0) & \equiv & R(1) & \pmod{p} \\
\mathbf{0} \times & (p_{n-1} 2^{n-1} + \cdots p_0) & \equiv & R(2) & \pmod{p} \\
 & & \vdots & & \\
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\end{array}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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Error locator polynomial: $E(x) = (x - e_1)$

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Error locator polynomial: $E(x) = (x - e_1)(x - e_2)$

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$E(i) = 0$ if and only if $e_j = i$ for some j

Where oh where can my bad packets be?

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod p \\
E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod p \\
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Multiply equations by $E(\cdot)$.

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All equations satisfied!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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Plugin points...

$$
\begin{aligned}
(p_2 + p_1 + p_0) &\equiv (3) && \pmod 7 \\
(4p_2 + 2p_1 + p_0) &\equiv (1) && \pmod 7 \\
(2p_2 + 3p_1 + p_0) &\equiv (6) && \pmod 7 \\
(2p_2 + 4p_1 + p_0) &\equiv (0) && \pmod 7 \\
(4p_2 + 5p_1 + p_0) &\equiv (3) && \pmod 7
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Error locator polynomial: $(x - 2)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$\begin{aligned}
(1-2)(p_2+p_1+p_0) &\equiv (3)(1-2) \pmod 7 \\
(2-2)(4p_2+2p_1+p_0) &\equiv (1)(2-2) \pmod 7 \\
(3-2)(2p_2+3p_1+p_0) &\equiv (6)(3-2) \pmod 7 \\
(4-2)(2p_2+4p_1+p_0) &\equiv (0)(4-2) \pmod 7 \\
(5-2)(4p_2+5p_1+p_0) &\equiv (3)(5-2) \pmod 7
\end{aligned}$$

Error locator polynomial: $(x-2)$.

Multiply equation i by $(i-2)$.

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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

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(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod{p} \\
&\vdots \\
(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod{p}
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$$Q(x) = a_{n+k-1}x^{n+k-1} + a_{n+k-2}x^{n+k-2} + \cdots a_0$$

Solving for $Q(x)$ and $E(x)$...

For all points $1, \ldots, i, n+2k$,

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a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
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a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod 7 \\
6a_3 + 2a_2 + 3a_1 + a_0 &\equiv 6(3 - b_0) \pmod 7 \\
a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
\end{aligned}$$

$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

$Q(x) = E(x)P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

$E(x) = x - b_0$

$Q(i) = R(i)E(i)$.

$$\begin{aligned}
a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod 7 \\
a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod 7 \\
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a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
\end{aligned}$$

$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

$Q(x) = x^3 + 6x^2 + 6x + 5$.

Example.

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\end{aligned}$$

$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

$Q(x) = x^3 + 6x^2 + 6x + 5$.

$E(x) = x - 2$.

Example: finishing up.

$$Q(x) = x^3 + 6x^2 + 6x + 5.$$

Example: finishing up.

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$E(x) = x - 2.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                1 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
          -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                    1 x^2 + 1 x + 1
          -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                    1 x^2 + 6 x + 5
                    1 x^2 - 2 x
                    ---------------
                              x + 5
                              x - 2
                              -----
                                  0
```

$P(x) = x^2 + x + 1$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                    1 x^2 + 1 x + 1
            -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                    1 x^2 + 6 x + 5
                    1 x^2 - 2 x
                    ---------------
                              x + 5
                              x - 2
                              -----
                                  0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                    1 x^2 + 1 x + 1
            -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                    1 x^2 + 6 x + 5
                    1 x^2 - 2 x
                    ---------------
                              x + 5
                              x - 2
                              -----
                                  0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                    1 x^2 + 1 x + 1
            -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                    1 x^2 + 6 x + 5
                    1 x^2 - 2 x
                    ---------------
                              x + 5
                              x - 2
                              -----
                                  0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1 Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                    1 x^2 + 1 x + 1
            -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                    1 x^2 + 6 x + 5
                    1 x^2 - 2 x
                    ---------------
                              x + 5
                              x - 2
                              -----
                                  0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1 Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where can my bad packets be?...

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

Proof:

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

Proof:
We claim

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Proof:
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Equation 2 implies 1:

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$

Unique solution for $P(x)$

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$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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$\implies Q'(x)E(x) = Q(x)E'(x)$.
Cross divide.

Unique solution for $P(x)$

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$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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We claim

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$\implies Q'(x)E(x) = Q(x)E'(x)$.
Cross divide. $\square$

Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

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Proof: Construction implies that

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Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$

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Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

Last bit.

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Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
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for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$.

Last bit.

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$$Q(i) = R(i)E(i)$$
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$$Q(i) = R(i)E(i)$$

$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

Last bit.

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When $E'(i)$ and $E(i)$ are not zero

Last bit.

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Last bit.

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points.

Last bit.

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Points to polynomials, have to deal with zeros!

Last bit.

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$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points. $\square$

Points to polynomials, have to deal with zeros!

Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Berlekamp-Welsh algorithm decodes correctly when k errors!

Summary. Error Correction.

Communicate n packets, with k erasures.

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Communicate n packets, with k erasures.

How many packets?

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Communicate n packets, with k erasures.

How many packets? $n+k$

Summary. Error Correction.

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How many packets? $n+k$
How to encode?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree?

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How many packets? $n+k$
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Of degree? $n-1$

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

Summary. Error Correction.

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

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How many packets? $n+k$
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Summary. Error Correction.

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
Why?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
Why?
k changes to make diff. messages overlap

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
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k changes to make diff. messages overlap
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How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
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How to encode? With polynomial, $P(x)$.

Summary. Error Correction.

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How to encode? With polynomial, $P(x)$. Of degree?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
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Recover? Reconstruct $P(x)$ with any n points!

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Reed-Solomon codes.

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Reed-Solomon codes. Welsh-Berlekamp Decoding.

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Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!