CS 70 Spring 2006 — Discrete Mathematics for CS, Vazirani — HW 3

1. Hard examples for stable marriage algorithm.

   (a) Run the traditional propose and reject algorithm on the following example.
   Men's preference list:

1	A	B	C	D
2	B	C	A	D
3	C	A	B	D
1	A	B	C	D

   Women's preference list:

A	2	3	4	1
B	3	4	1	2
C	4	1	2	3
D	1	2	3	4

   (b) In class we showed that the propose and reject algorithm must terminate after at most n^2 proposals. Prove a sharper bound showing that the algorithm must terminate after at most $n(n-1)+1$ proposals. Conclude that the above example is a worst case instance for $n=4$. How many days does the algorithm take on this instance?

   (c) **Extra credit:** generalize the above example to arbitraty n and prove rigorously that the algorithm makes $n(n-1)+1$ proposals on your example. How many days does the algorithm take?

2. Suppose we relax the rules for the men, so that each unpaired man proposes to the next woman on his list at a time of his choice (some men might procrastinate for several days, while others might propose and get rejected several times in a single day). Can the order of the proposals change the resulting pairing? Give an example of such a change or prove that the pairing that results is the same.

3. Give a proof by induction that in the game from lecture 1, a player who receives an even number has a winning strategy.

4. A line divides a plane into two regions. Two lines divide it into four regions (unless they are parallel in which case it is three regions. In this question we are interested in the maximum number of regions so we won't consider this case). Prove by induction on n that the maximum number of regions that the plane is divided into by n lines is $\frac{n^2+n+2}{2}$.

   Hint: How many times can the n-th line intersect the remaining lines, and how many new regions does each such intersection result in?