Problem Set #10

1. **Total Annihilation**
   A super power has 2620 missiles stored in well separated silos. An enemy is considering a sneak attack. However, for the attack to succeed every one of the missiles must be destroyed. Assume that each attacking warhead hits one of the enemy missiles with each enemy missile being equally likely to be the one that is hit. How many warheads on the average will be needed to ensure the complete destruction of every enemy missile?

2. **Casino wins**
   A gambler plays 120 hands of draw poker, 60 hands of black jack, and 20 hands of stud poker per day. He wins a hand of draw poker with probability 1/6, a hand of black jack with probability 1/2, and a hand of stud poker with probability 1/5. Assume the outcomes of the card games are mutually independent.

   (a) What is the expected number of hands the gambler wins in a day?

   (b) What is the variance in the number of hands won per day?

   (c) What would the Markov bound be on the probability that the gambler will win 108 hands on a given day?

   (d) What would the Chebyshev bound be on the probability that the gambler will win 108 hands on a given day?

3. **Find the right key**
   A man has a set of n keys, one of which fits the door to his apartment. He tries the keys until he finds the correct one. Give the expected number and variance for the number of trials until success if

   (a) he tries the keys at random (possibly repeating a key tried earlier).

   (b) he chooses keys randomly from among those he has not yet tried.

4. **The Martingale**
   Consider a *fair game* in a casino: on each play, you may stake any amount $\$X$; you win or lose with probability $\frac{1}{2}$ each (all plays being independent); if you win you get your stake back plus $\$X$; if you lose, you lose your stake.

   (a) What is the expected number of plays before your first win (including the play on which you win)?

   (b) The following gambling strategy, known as the "martingale," was popular in European casinos in the 18th century: on the first play, stake \$1; on the second play \$2; on the third play \$4; and in general, on the kth play $\$2^{k-1}$. Stop (and leave the casino!) when you first win. Show that if you follow this strategy, and assuming you have unlimited funds available, then you will leave the casino \$1 richer with probability 1. (Maybe this is why the strategy is banned in most modern casinos).

   (c) To discover the catch in this seemingly infallible startegy, let X be the random variable that measures your maximum loss before winning (i.e., the amount of money you have lost *before* the play on which you win.) Show that $\mathbf{E}[X] = \infty$. What does this imply about your ability to play the martingale strategy in practice?