Problem Set 2

1. **Simple Induction** (5 pts)
   Prove that for every $n \geq 2$, $\sqrt{1+\sqrt{1+\ldots\sqrt{1}}}$ (n square roots) is irrational.

2. **Recursion and proofs** (5 pts)
   Let the function, g, be defined on the natural numbers recursively as follows: $g(0) = 0$, $g(1) = 1$, and $g(n) = 5g(n-1) - 6g(n-2)$, for $n \geq 2$.
   Prove that $\forall n \in \mathbb{N}, g(n) = 3^n - 2^n$.

3. **Making big rocks into little rocks** (5 pts)
   One day, you find a shady-looking flyer on Sproul that advertises an undergraduate research position in the Stanford geology department. After you make your way down to the Farm, Dr. Hoover, the professor in charge, hands you the following assignment to keep you busy for the first year, complaining that none of the Stanford undergrads have been able to help him:

   You are given a single large pile of pebbles to start with. Every day, you have pick one pile of pebbles and split it into two smaller piles any way you wish. Then, you have to count the number of pebbles in each of the two new piles you just created (k and m), and write down the product $k \times m$ in your lab notebook. Of course, over time, you end up with more and more piles which get smaller and smaller. You get to stop when each pebble is in its own pile. Once you stop, add up all the numbers you wrote down and report them to Dr. Hoover. See Figure 1 for an example of the process.

   Convince Dr. Hoover (and his funding agency) that this[1] is a pointless exercise: no matter how you split the piles, your answer will come out to $n(n-1)/2$, where n is the number of pebbles in the original pile (which you can just count on the first day).

4. **Stable Marriage (aka stable matching)**

   (a) (5 pts) For the following instance of the stable marriage problem:

Man	highest→lowest		
1	B	A	C
2	C	B	A
3	A	C	B

Table 1: Men's preference list

Woman	highest→lowest		
A	1	2	3
B	2	3	1
C	3	1	2

Table 2: Women's preference list

   i. List the rogue couples in the pairing $(1, C), (2, B), (3, A)$.
   ii. List **all** the possible stable pairings (note that a single run of the algorithm from class will only reveal one; there may be others).

   (b) (3 pts) Run the traditional propose and reject algorithm on the following example:

Man	highest→lowest			
1	A	B	C	D
2	B	C	A	D
3	C	A	B	D
4	A	B	C	D

Table 3: Men's preference list

Woman	highest→lowest			
A	2	3	4	1
B	3	4	1	2
C	4	1	2	3
D	1	2	3	4

Table 4: Women's preference list

[1] The research assignment, not this homework problem!

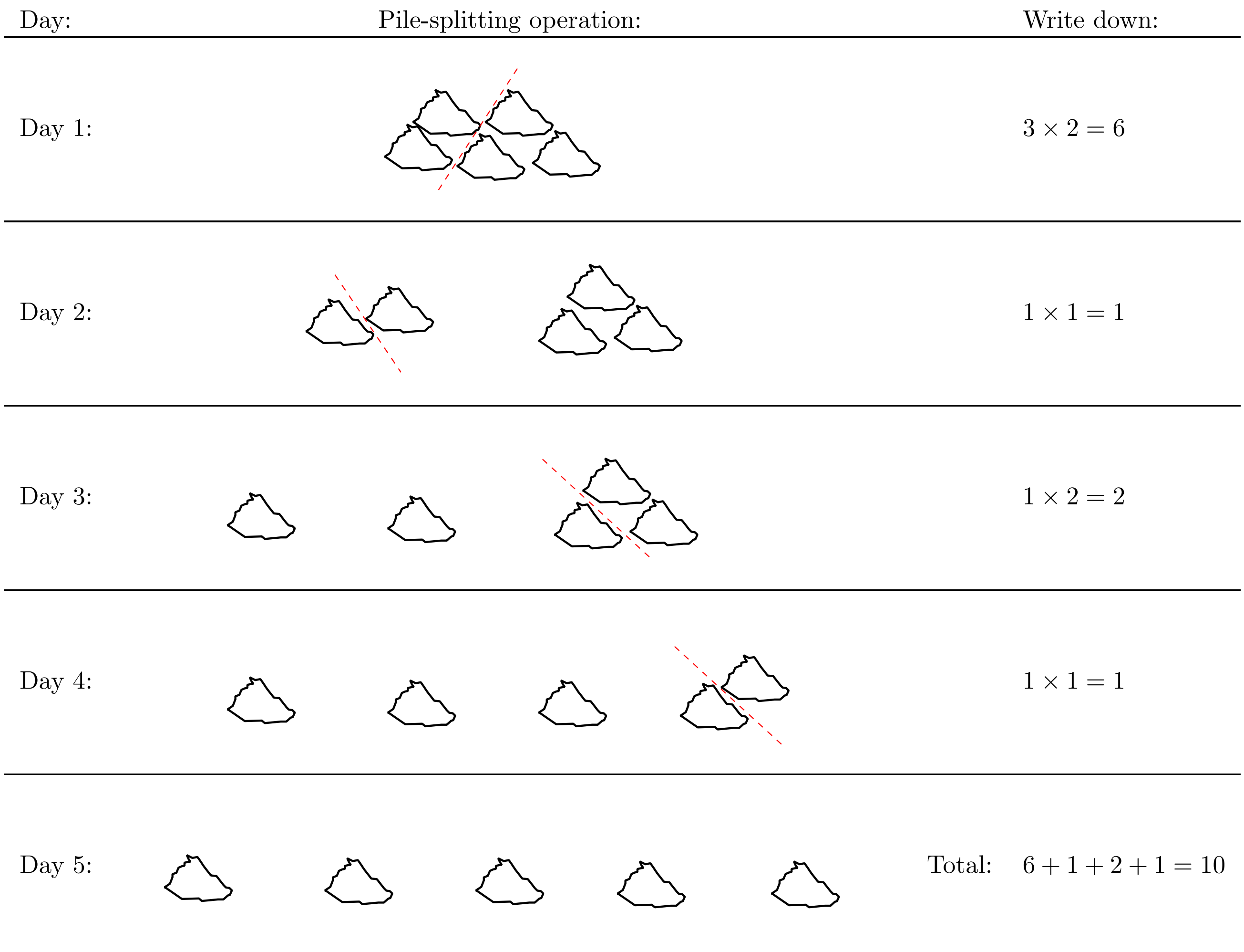

Figure 1: An example rock procedure for Problem 3 with $n = 5$. Bonus (0 pts): what are those rocks?