Problem Set 9

1. **Sum the Poissons**
   Let λ_x and λ_y to be real numbers. $X \sim \text{Poi}(\lambda)$ denotes that random variable X has a Poisson dirstribution with mean of λ. Suppose $X \sim \text{Poi}(\lambda_x)$ and $Y \sim \text{Poi}(\lambda_y)$ are independent random variables. Show that $X + Y \sim \text{Poi}(\lambda_x + \lambda_y)$.

2. **Machine Failures**
   Two faulty machines, M_1 and M_2, are repeatedly run synchronously in parallel (i.e., both machines execute one run, then both execute a second run, and so on). On each run, M_1 fails with probability p_1 and M_2 with probability p_2, all failure events being independent. Let the random variables X_1 and X_2 denote the number of runs until the first failure of M_1,M_2 respectively; thus X_1 and X_2 have geometric distributions with parameters p_1, p_2 respectively.

   Let $X = \min\{X_1, X_2\}$ denote the number of runs until the first failure of *either* machine. Show that X also has a geometric distribution, with parameter $p_1 + p_2 - p_1 p_2$.

3. ***Poisson*ed Misprints**
   A textbook has on average one misprint per page. What is the chance that you see exactly 4 misprints on page 1? What is the chance that you see exactly 4 misprints on some page in the textbook of 250 pages? (Hint: Model the number of misprints on a single page by a Poisson.)

4. **Multiple Choice**
   A multiple-choice quiz has 100 questions each with four possible answers of which only one is correct. What is the probability that sheer guesswork yields between 10 and 30 correct answers for the 40 of the 100 problems about which the student has no knowledge? (You can leave your answer in the summation form.)

5. **Coin Distributions**
   Suppose you flip n fair, independent coins. Let the random variable X be the number of heads that come up.

   (a) What is the exact value of $\Pr(X \leq k)$, the probability of flipping k or fewer heads? Your answer need not be in closed form.

   (b) Suppose $k < n/2$. Prove that:

   $$\Pr(X \leq k) \leq \frac{n-k+1}{n-2k+1} \cdot \Pr(X = k)$$

   (Upper bound your previous answer with an infinite geometric sum and then evaluate the sum.)

   (c) If you flip a coin 100 times, the probability of flipping exactly 30 heads is approximately 23 out of a million. Give an upper bound on the probability of flipping 30 or *fewer* heads.