EECS 70 Discrete Mathematics and Probability Theory
Spring 2014 Anant Sahai

Discussion 10B

1. **Prove!**

   Consider the following identity:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

   (a) Prove the identity by algebraic manipulation.

   (b) Prove the identity using a combinatorial argument.

   Consider the following identity:

$$\binom{2n}{2} = 2\binom{n}{2} + n^2.$$

   (a) Prove the identity by algebraic manipulation.

   (b) Prove the identity using a combinatorial argument.

2. **Permutations**

Consider n people. Each person has a role model among the n people (which could be the person herself/himself) and it turns out that all role models are unique. In other words the role models form a bijection from the set of n people to themselves, which is also called a permutation. In this permutation, we can consider cycles. If we start from a person A, then her role model is going to be a person B who also has a role model C and so on.

- Prove that if we continue following the chain of role models this way, we have to return to A before we meet any other repetition.

- The length of a cycle is an interesting number. Fix a person A and a number k. How many different permutations result in that person being in a cycle of length k? If all permutations were equality likely, what is the corresponding probability (i.e. the count divided by the total number of permutations)?

3. **Balls and bins**

You have n bins and you throw balls into them one by one randomly. A collision is when a ball is thrown into a bin which already has another ball.

(a) What is the probability that the first ball thrown will cause the first collision?

(b) What is the probability that the second ball thrown will cause the first collision?

(c) What is the probability that, given the first two balls are not in collision, the third ball thrown will cause the first collision?

(d) What is the probability that the third ball thrown will cause the first collision?

(e) What is the probability that, given the first $m-1$ balls are not in collision, the mth ball thrown will cause the first collision?

(f) What is the probability that the mth ball thrown will cause the first collision?