EECS 70 Discrete Mathematics and Probability Theory
Spring 2014 Anant Sahai

Discussion 12B

1. **Locked out**
   You just rented a large house and the realtor gave you five keys, one for the front door and the other four for each of the four side and back doors of the house. Unfortunately, all keys look identical, so to open the front door, you are forced to try them at random.

   Find the distribution and the expectation of the number of trials you will need to open the front door. (Assume that you can mark a key after you've tried opening the front door with it and it doesn't work.)

2. **A roll of the dice**
   Consider a single roll of two dice, one red and one blue.

   (a) Let R be the value of the red die. What is the distribution of R? What is the expectation of R?

   (b) Let M be the maximum of the numbers on the two dice. What is the distribution of M? What is the expectation of M?

   (c) How do the distribution and expectation of M compare to that of R?

3. **To pay or not to pay?**
   Alice goes to Berkeley and she drives to school everyday. Tired of always paying for parking, Alice decides one day not to pay her parking fees. Assume that there is a probability of 0.05 that she gets caught by the meter maid. The parking fee is \$0.25 and if she is caught, her parking ticket is \$10.

   (a) How does the expected cost of parking 10 times without paying the meter compare with the cost of paying the meter each time? (*Hint*: Think of Alice getting caught or not as a single biased coin flip with probability 0.05.)

   (b) If she parks at the meter 10 times, what is the probability that she will have to pay more than the total amount he could end up saving by not putting the money?

4. **Prove it**
Assume the random variable X takes on integer values from $1, 2, \ldots n-1, n$. Prove that

$$E[X] = \sum_{i=1}^{n} P(X \geq i).$$

5. **Random variables modulo p**
Let the random variables X and Y be distributed independently and uniformly at random in the set $\{0, 1, \ldots, p-1\}$, where $p > 2$ is a prime.

   (a) What is the expectation $E[X]$?

   (b) Let $S = (X+Y) \bmod p$. What is the distribution of S?

   (c) What is $E[S]$?

   (d) By linearity of expectation, we might expect that $E[S] = (E[X] + E[Y]) \bmod p$. Explain why this does not hold in the present context.