1. Strengthening the induction hypothesis

Suppose we have a supply of L-shaped tiles (2x2 tiles with a 1x1 square removed), and a $2^n \times 2^n$ grid ($n \geq 1$), prove that we can cover the grid with tiles such that only one square is left uncovered.

2. Recursion and induction

Working at the local pizza parlor, I have a stack of unbaked pizza doughs. For a most pleasing presentation, I wish to arrange them in order of size, with the largest pizza on the bottom. I know how to place my spatula under one of the pizzas and flip over the whole stack above the spatula (reversing their order). This is the only move I know that can change the order of the stack; however, I am willing to keep repeating this move until I get the stack in order. Give a recursive procedure to get the pizzas in order. Prove by induction that your procedure correctly orders the pizzas.

3. More Induction

Suppose that I start with 0 written on a piece of paper. Each minute, I choose a digit written on the paper and erase it. If it was 0, I replace it with 010. If it was 1, I replace it with 1001. Prove that no matter which digits I choose and no matter how long the process continues, I never end up with two 1's in a row.

4. Dividing n-gon

Assume that any simple (but not necessarily convex) n-gon ($n > 3$) has a diagonal (line between two non-adjacent vertices) that lies completely within the n-gon. Show that any such n-gon ($n \geq 3$) can be divided into $n-2$ triangles such that all vertices of each triangle are vertices of the n-gon.

5. Fibonacci Sums

The sequence of Fibonacci numbers is defined by: $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ if $n \geq 2$. Thus the sequence starts with 1, 1, 2, 3, 5, 8, 13 ... While not all natural numbers are Fibonacci numbers, interestingly, every natural number can be written as the sum of different Fibonacci numbers. Prove this result. (Hint: Use strong induction.)

6. False Proof

What goes wrong in the following "proof"?

Theorem: If n is an even number and $n \geq 2$, then n is a power of two.

Proof:
By induction on the natural number n. Let the induction hypothesis $IH(k)$ be the assertion that "if k is an even number and $k \geq 2$, then $k = 2^i$, where i is a natural number".

Base case: $IH(2)$ states that 2 is a power of two, which it is ($2 = 2^1$).

Inductive step: Assume that k is a number greater than 2, and that $IH(j)$ holds for all $2 \leq j < k$.

Case 1: k is odd, and there is nothing to show.

Case 2: k is even, so $k \geq 4$. Since $k \geq 4$ is an even number, $k = 2l$, with $2 \leq l < k$. Therefore we can use the induction hypothesis $IH(l)$, which asserts that $l = 2^i$ for some integer i. Thus we have $k = 2l = 2^{i+1}$, so k is a power of two. $IH(k)$ holds.