Section 7

Wednesday, July 17

CS 70: Discrete Mathematics and Probability Theory, Summer 2013

1. These problems will help you understand hypercubes.

   1a. Show that the edges of an n-dimensional hypercube can be colored using n colors so that no pair of edges sharing a common vertex have the same color.

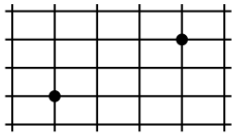
   1b. Show that the vertices of an n-dimensional hypercube can be colored using 2 colors so that no pair of adjacent vertices have the same color. (This is equivalent to showing that a hypercube is *bipartite*: the vertices can be partitioned into two groups (according to color) so that every edge goes between the two groups.)

2. See if you can use the balls and bins framework to solve these problems.

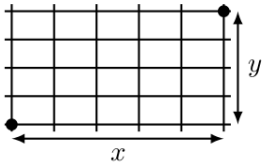
   2a. In how many ways can 10 cans of Pepsi be distributed among three people A, B, C?

   2b. In how many ways can 20 cans of Pepsi and 5 cans of Coke be distributed among four people A, B, C, D?

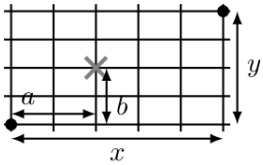
3. Manhattan is well-known for its grid layout and busy traffic, so you are interested in evaluating different shortest paths from one point to another.



Part (b)



Part (c)



Part (d)

   3a. Formally define the graph of intersections between the 20th street and the 60th street, and between the 1st avenue and the 10th avenue (assuming a perfect grid layout). That is, specify the set of vertices and the set of edges using mathematical notation.

   3b. Consider two locations that are 3 blocks by 2 blocks away from each other. If you take a cab from one point to the other, how many shortest paths are there?

   3c. How many shortest paths are there connecting two points that are x blocks by y blocks away from each other? (x and y are non-negative integers.)

   3d. Assume that an intersection, a blocks by b blocks away from the starting point, is under road work and is not usable. How many shortest paths are there connecting the starting point and the destination without using that intersection (which is assumed to be between the starting point and the destination, see figure above)?