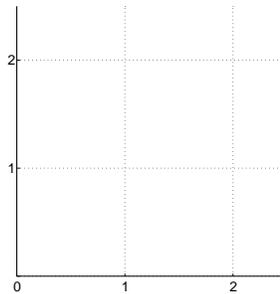


1. Visualizing Error Correction

Alice wants to send a message of 2 packets to Bob, and wants to guard against 1 lost packet. So working over $GF(3)$, she finds the unique polynomial $P(x)$ that passes through the points she wants to send, and sends Bob her augmented message of 3 packets: $(0, P(0)), (1, P(1)), (2, P(2))$.

One packet is lost, so Bob receives the following packets: $(0, 2), (2, 0)$.

- (a) Plot the points represented by the packets Bob received on the grid below.



- (b) Draw in the unique polynomial $P(x)$ that connects these two points.

- (c) By visual inspection, find the lost packet $(1, P(1))$.

2. Erasure Warm-Up

Working over $GF(q)$, you want to send your friend a message of $n = 4$ packets and guard against 2 lost packets. What is the minimum q you can use? What is the maximum degree of the unique polynomial that describes your message?

3. Aliens, Oh My!

Alice wants to send a plea for help to an alien space ship that is hovering near her city. She knows that at their current distance of 7 miles above ground, no more than 3 general errors can occur during transmission. If she sends a message of length 15, how long must her original message be?

4. Where Are My Packets?

Alice wants to send the message (c_0, c_1, c_2) to Bob, where each $c_i \in \{0, 1, 2, 3, 4\}$. She encodes it as a polynomial P of degree ≤ 2 over $GF(5)$ such that $P(0) = c_0$, $P(1) = c_1$, and $P(2) = c_2$, and she sends the packets $(0, P(0))$, $(1, P(1))$, $(2, P(2))$, $(3, P(3))$, $(4, P(4))$. Two packets are dropped, and Bob only learns that $P(0) = 4$, $P(3) = 1$, and $P(4) = 2$. Help Bob recover Alice's message.

- (a) Find the multiplicative inverses of 1, 2, 3 and 4 modulo 5.
- (b) Find the original polynomial P by using Lagrange interpolation or by solving a system of linear equations.
- (c) Recover Alice's original message.

5. More Erasures!

Consider the alphabet $A = 0$, $B = 1$, $C = 2$, $D = 3$, $E = 4$. Suppose a message of length 3 is sent using the error correction scheme discussed in class over $GF(5)$. If you receive the following packets, what was the original message?

- (a) $C _ A A$
- (b) $_ A C C$
- (c) Can you determine the original message if you only receive $C E _ _$? Either find the original message or explain why you can't.