CS 70 Discrete Mathematics and Probability Theory
Summer 2015 Chung-Wei Lin Discussion 4A

1. Balls and Bins

How many ways to put 3 balls into 5 bins?

(a) If balls are distinguishable and bins are distinguishable?

(b) If balls are undistinguishable and bins are distinguishable?

(c) If balls are undistinguishable and bins are undistinguishable?

(d) If balls are distinguishable and bins are undistinguishable?

Answer:

(a) $5^3 = 125$.

(b) $\binom{7}{3} = 35$.

(c) 3 (brute-force).

(d) 5 (brute-force, Stirling numbers).

2. Balls from a Bag

How many ways to select 3 balls from a bag in which there are 5 different balls?

(a) If order does matter with sampling with replacement?

(b) If order does matter with sampling without replacement?

(c) If order does not matter with sampling without replacement?

(d) If order does not matter with sampling with replacement?

Answer:

(a) $5 \times 5 \times 5 = 125$.

(b) $5 \times 4 \times 3 = 60$.

(c) $\binom{5}{3} = 10$.

(d) $\binom{7}{3} = 35$.

3. Combinatorial Proof

Prove the following using combinatorial arguments.

(a) $\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{k} \cdot \binom{n}{r-k}$.

(b) $k\binom{n}{k} = n\binom{n-1}{k-1}$.

Answer:

(a) Choose r elements from two sets which have m and n elements, respectively.

(b) Choose k elements from n elements and choose one from the k elements.

4. License Plate

A license plate contains 7 characters (order matters). Each character may either be an upper-case letter A–Z or a number 0–9. How many license plates...

(a) contain only letters?

(b) have exactly three letters and four numbers?

(c) contain the string ABC?

(d) have at least two of the same character?

Answer:

(a) 26^7.

(b) $\binom{7}{3}26^3 10^4$.

(c) $\binom{5}{1}36^4 - 3 \times 36$. Note: $\binom{5}{1}36^4$ counts each of the following cases twice: ABCABC_, ABC_ABC, and _ABCABC .

(d) This is the total number of license plates minus the ones that have all different characters: $36^7 - \frac{36!}{29!}$.