CS 70 Discrete Mathematics and Probability Theory
Summer 2015 Chung-Wei Lin Discussion 5D

Definition: The expectation of a discrete random variable X is defined as $\Sigma_{a\in\mathbb{A}} a \times Pr[X=a]$ where the sum is over all possible values taken by the r.v. *Intuitively, this is the average value, weighted by the probability.*

1. Welcome to the dungeon

(a) You're playing a simple game of DnD, playing with a 6 sided die. Let X be the number that comes up top. What is the expected value of X?
$\frac{7}{2}$.

(b) What is $\mathbb{E}(X^2)$? What is $(\mathbb{E}(X))^2$? Are they ever equal?
$\mathbb{E}(X^2) = \frac{91}{6}$, $\mathbb{E}(X)^2 = \frac{49}{4}$. They're equal when X only has one value.

(c) Unfortunately, the mechanics of DnD make it a bit more complicated than just a simple roll. If you roll a 1, you deal no damage. However, if you roll a 6, you deal 12 damage instead of 6. What is $\mathbb{E}(X)$ now?
$\frac{13}{3}$.

2. Can you even math?

For each of the following, either prove the assertion or find a counterexample.

(a) $\mathbb{E}(\alpha X) = \alpha\mathbb{E}(X)$.
See Note 12.

(b) $\mathbb{E}(X+Y) = \mathbb{E}(X) + \mathbb{E}(Y)$.
See Note 12.

(c) $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y)$.
If $\mathbb{E}(X) = 0$ but X is nonzero, then finding a Y is trivial.

(d) $\mathbb{E}(\frac{1}{X}) = \frac{1}{\mathbb{E}(X)}$.
$X = ((-1, 0.5), (1, 0.5))$

3. Do you want to play a game?

(a) You're at a carnival, where one of the booths has the following game: You start with a prize pot of 2 dollars. On every turn, if you flip a coin. If it comes up heads, you double the pot (i.e. 2 dollars becomes 4). If it comes up tails, you take the money in the pot and walk. What's the probability distribution?

$\{(2^x, \frac{1}{2^x}) | x \in N^+\}$.

(b) What's the expected return?

The return is ∞.

(c) How could you modify the coin to make the expected value finite?

$\mathbb{E}(X)$ will be finite if $\Pr[head] < 0.5$.

4. Midterms are going to kill us all

When solving a midterm problem, you start by trying an approach, which is is right with probability p and wrong with probability $1 - p$. If your approach is right, then you'll solve it in 2 minutes. If your approach is wrong, you'll realize that you made a mistake after 3 minutes and have to go back to the approach selection step. Hard problems have $p = 0.1$, medium problems have $p = 0.5$, and easy problems have $p = 0.9$.

(a) What is the expected amount of time you spend on each problem?

$29, 5, \frac{7}{3}$. Note: it is $2 + \frac{3(1-p)}{p}$.

(b) If you increase each of your p values by 0.1, how much time do you need on each category?

$14, 4, 2$.